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Research Article

FEM ANALYSIS OF THE INFLUENCE OF THE PLATE GEOMETRY ON THE BUCKLING DELAMINATION OF A PIEZOELECTRIC SANDWICH RECTANGULAR THICK PLATE

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ABSTRACT

In this study, we attempt to investigate the influence of the plate geometry on a buckling delamination of a PZT/Metal/PZT sandwich rectangular thick plate with lengths ℓ_1 and ℓ_3, and with thickness *h*. We assume that between the face and core layers there are two parallel interface-band cracks. We also assume that the considered thick plate is mechanically simply supported and unelectroded with vanishing normal electric displacement on its four lateral edges and, ideal contact conditions are satisfied between the interfaces of the contact layers. This plate is compressed only two lateral surfaces by the uniform uniaxial normal forces and there are neither mechanically nor electrically forces act on the upper and lower surfaces and also cracks' surfaces of the rectangular sandwich plate. The considered boundary value problems is modeled mathematically within the scope of the exact geometrically nonlinear equations of the theory of electro-elasticity within the scope of the piecewise homogeneous body model and is solved numerically by using 3D-FEM. The influence of various material and geometric parameters as well as the couple effect between the electrical and the mechanical fields on the buckling delamination of the rectangular thick plate is analyzed and discussed.

Keywords: Piezoelectric sandwich rectangular thick plate, buckling-delamination, FEM.

1. INTRODUCTION

Elastic materials that have an interaction between the mechanical and the electrical fields are called electro-elastic materials. If this couple effect between these two fields is linear, these electro-elastic materials are specifically called piezoelectric materials which were discovered experimentally by the Curie brothers in the 1880s. These materials exhibit electrical polarization when mechanical force applied or, conversely, exhibit mechanical deformation when placed in the electrical field. According to favorable properties of these materials, they are used in many engineering applications, for example, sensors used to detect very small/sensitive effects (the change of pressure, current, temperature etc.), piezoelectric generator (electric harvesting) and so

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on. These materials that widely used in many engineering applications, are the subjects of many theoretical and experimental scientific researches. The most common usage of piezoelectric materials (PZT) in th e component of the structures is as plates that is agglutinated to the base component. As a result of this process, some non-stick surfaces such as crack-like can occur between the PZT plate and the base construction due to various technological processes or inconvenient conditions. It is evident that these crack-like voids significantly affect strength of the structural elements under various external effects during their usage life, and cause unacceptable experiences such as fracture and buckling delamination of the mentioned structural elements. Some theoretical researches made on these problems are summarized below.

In References [1, 2] the buckling delamination problems were modelled mathematically in the scope of the three dimensional geometrically nonlinear exact equations of the theory of viscoelasticity, and studied for the elastic and/or viscoelastic rectangular thick plates with rectangular cracks under some boundary and loading conditions. These mathematical models and boundary value problems arising within selected boundary conditions were solved numerically by means of three-dimensional finite element modeling (3D-FEM) and critical parameter values were determined for various materials and geometrical parameters.

In the papers [3, 4] the buckling-delamination problems around the interface cracks between the layers of the PZT/Metal/PZT sandwich plate (for a plate-strip [3] in the plane-strain state (2D) and for a rectangular thick plate in 3D [4]) under given boundary and loading conditions are studied. In these studies, the effects of various geometrical and material parameters and also interaction between the electrical and the mechanical fields on the critical buckling-delamination forces are modeled and studied in the context of the nonlinear exact equations of the electro-elasticity theory and the piecewise homogeneous body model. According to the numerical results of this study, it is established that the influence of the couple effects between the mechanical and the electrical fields on the values of the critical buckling delamination force is significant.

It should be noted that the all numerical investigations carried out in the paper [4] are made only for a fixed value of the geometrical parameter which characterizes the ratio of the plate length in the Ox_1 and Ox_3 axes (Fig. 1). It is also should be noted that namely through this parameter it is possible to estimate the influence of the three-dimensionality of the problem on the critical values of the external compressive force. Taking this situation into consideration in the present paper the numerical investigations started in the paper [4] is continued for the various values of the aforementioned parameter.

As in the paper [4], the investigations are made within the framework of the geometric nonlinear exact equations of the three-dimensional theory of electro-elasticity for piezoelectric materials and solved numerically by employing of the three-dimensional finite element formulation. All the algorithms and PC programs required for the numerical solutions are composed by the authors. Numerical results illustrated the influence of the aforementioned geometrical parameter on the critical values of the external forces are presented and discussed for various PZT materials and for the various values of the geometrical parameter characterizing the cracks' length in the Ox_1 axis direction.

2. FORMULATION OF THE PROBLEMS

Assume that the considered piezoelectric sandwich rectangular thick plate has $\ell_1 \times h \times \ell_3$ dimensions along the Ox_1, Ox_2 and Ox_3 axes respectively and subjected to the uniform external compressive forces whose density is p as given in Fig.1a.

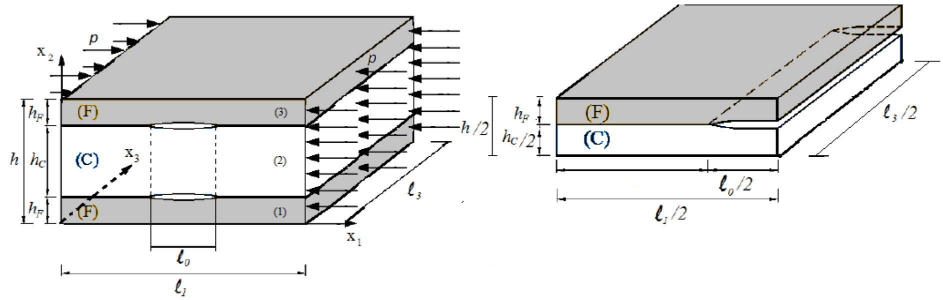

Figure 1. The geometry of the plate and the considered external forces; solution domain (a), one-eighth of the region (b)

Solution domain of the problem studied is

$$\Omega = \Omega_1 \cup \Omega_2 \cup \Omega_3 - S_L^{\pm} - S_U^{\pm} \tag{1}$$

where

$$\begin{aligned}
&\Omega_1 = \{0 \le x_1 \le \ell_1; 0 \le x_2 \le h_F; 0 \le x_3 \le \ell_3\},\\
&\Omega_2 = \{0 \le x_1 \le \ell_1; h_F \le x_2 \le h_F + h_C; 0 \le x_3 \le \ell_3\},\\
&\Omega_3 = \{0 \le x_1 \le \ell_1; h_F + h_C \le x_2 \le 2h_F + h_C; 0 \le x_3 \le \ell_3\},\\
&S_L^{\pm} = \{(\ell_1 - \ell_0)/2 \le x_1 \le (\ell_1 + \ell_0)/2; x_2 = h_F \mp 0; 0 \le x_3 \le \ell_3\},\\
&S_U^{\pm} = \{(\ell_1 - \ell_0)/2 \le x_1 \le (\ell_1 + \ell_0)/2; x_2 = h_F + h_C \mp 0; 0 \le x_3 \le \ell_3\}.
\end{aligned} \tag{2}$$

Here, Ω_1, Ω_2 and Ω_3 represents bottom, middle and top layers, respectively; $S_L^{\pm}(S_U^{\pm})$ represents the lower and upper surfaces of the band crack between the bottom and the middle (middle and top) layers. Governing field equations provided in these regions, for each r_n-th (n=1,2,3) layer, are given below the framework of the three dimensional geometrically nonlinear exact equations of the electro-elasticity,

$$\begin{aligned}
&\frac{\partial K_{ji}^{(r_n)}}{\partial x_j} = 0, \frac{\partial D_j^{(r_n)}}{\partial x_j} = 0,\\
&K_{ji}^{(r_n)} = T_{jk}^{(r_n)}\left(\delta_i^k + \frac{\partial u_i^{(r_n)}}{\partial x_k}\right) + M_{ji}^{(r_n)}, D_i^{(r_n)} = e_{ikl}^{(r_n)} s_{kl}^{(r_n)} + \varepsilon_{ik}^{(r_n)} E_k^{(r_n)}\\
&T_{ij}^{(r_n)} = c_{ijkl}^{(r_n)} s_{kl}^{(r_n)} - e_{kij}^{(r_n)} E_k^{(r_n)}, s_{kl}^{(r_n)} = \frac{1}{2}\left(\frac{\partial u_k^{(r_n)}}{\partial x_l} + \frac{\partial u_l^{(r_n)}}{\partial x_k} + \frac{\partial u_i^{(r_n)}}{\partial x_l}\frac{\partial u_i^{(r_n)}}{\partial x_k}\right)\\
&M_{ji}^{(r_n)} = \varepsilon_0\left(E_i^{(r_n)} E_j^{(r_n)} - \frac{1}{2} E_k^{(r_n)} E_k^{(r_n)} \delta_i^j\right), E_k^{(r_n)} = -\frac{\partial \phi^{(r_n)}}{\partial x_k}, \ i, j, k, n = 1, 2, 3
\end{aligned} \tag{3}$$

In (2) the following notation is use: T_{jk} is the component of the ordinary stress tensor; M_{ji} is the component of Maxwell stress tensor; D_i is the component of the electrical displacement vector; e_{ikl} is the piezoelectric constant; S_{kl} is the component of Green strain tensor; ε_{kl} is the dielectric constant; E_k is the component of the electrical field vector; c_{ijkl} is the elastic constant and ϕ is the electric potential. Also ε_0 is the permittivity of free space and δ_i^j is the Kronecker symbol.

Assume that all the lateral surfaces of the considered plate are simply supported and with zero electrical potential and the plate is subjected to static external compressive forces only act on $x_1=0;\ell_1$ ends. It is also assumed that the mechanical forces and electrical charges do not affect on the bottom/top and cracks' surfaces of the plate. Accordingly, mathematical expressions of these boundary conditions can be formulated as follows,

$$
\begin{aligned}
&u_2^{(r_n)}\big|_{x_1=0;\ell_1}=0,\ \phi^{(r_n)}\big|_{x_1=0;\ell_1}=0,\\
&u_2^{(r_n)}\big|_{x_3=0;\ell_3}=0,\ \phi^{(r_n)}\big|_{x_3=0;\ell_3}=0,\\
&K_{11}^{(r_n)}\big|_{x_1=0}=K_{11}^{(r_n)}\big|_{x_1=\ell_1}=p,\\
&K_{13}^{(r_n)}\big|_{x_1=0;\ell_1}=K_{31}^{(r_n)}\big|_{x_3=0;\ell_3}=K_{33}^{(r_n)}\big|_{x_3=0;\ell_3}=0,\\
&K_{2i}^{(r_3)}\big|_{x_2=h}=K_{2i}^{(r_1)}\big|_{x_2=0}=0,\\
&D_2^{(r_1)}\big|_{x_2=0}=D_2^{(r_3)}\big|_{x_2=h}=0,\\
&K_{ji}^{(r_3)}\big|_{S_U^+}n_{jU}^{+}=K_{ji}^{(r_2)}\big|_{S_U^-}n_{jU}^{-}=D_j^{(r_3)}\big|_{S_U^+}n_{jU}^{+}=D_j^{(r_2)}\big|_{S_U^-}n_{jU}^{-}=0,\\
&K_{ji}^{(r_2)}\big|_{S_L^+}n_{jL}^{+}=K_{ji}^{(r_1)}\big|_{S_L^-}n_{jL}^{-}=D_j^{(r_2)}\big|_{S_L^+}n_{jL}^{+}=D_j^{(r_1)}\big|_{S_L^-}n_{jL}^{-}=0
\end{aligned}
\tag{4}
$$

For $0\le x_1\le(\ell_1-\ell_0)/2\cup(\ell_1+\ell_0)/2\le x_1\le\ell_1$ and $0\le x_3\le\ell_3$, the contact conditions between the layers of sandwich plate are as follows,

$$
\begin{aligned}
&K_{2i}^{(r_1)}\big|_{x_2=h_F}=K_{2i}^{(r_2)}\big|_{x_2=h_F},\ u_i^{(r_1)}\big|_{x_2=h_F}=u_i^{(r_2)}\big|_{x_2=h_F}\\
&\phi^{(r_1)}\big|_{x_2=h_F}=\phi^{(r_2)}\big|_{x_2=h_F},\ D_2^{(r_1)}\big|_{x_2=h_F}=D_2^{(r_2)}\big|_{x_2=h_F}\\
&K_{2i}^{(r_2)}\big|_{x_2=h_F+h_C}=K_{2i}^{(r_3)}\big|_{x_2=h_F+h_C},\ u_i^{(r_2)}\big|_{x_2=h_F+h_C}=u_i^{(r_3)}\big|_{x_2=h_F+h_C}\\
&\phi^{(r_2)}\big|_{x_2=h_F+h_C}=\phi^{(r_3)}\big|_{x_2=h_F+h_C},\ D_2^{(r_2)}\big|_{x_2=h_F+h_C}=D_2^{(r_3)}\big|_{x_2=h_F+h_C}
\end{aligned}
\tag{5}
$$

The boundary value problem (1)-(4) represents the buckling delamination of a piezoelectric rectangular sandwich thick plate, whose face layers are made of piezoelectric material and the core layer is made of metal material and which has two parallel band cracks between its layers under uniformly distributed static external compressive forces acting at two opposite lateral surfaces. This problem is a nonlinear one and the solution to this problem is reduced to the solution of series linear boundary value problems as a result of linearization procedures [5].

This reducing is based on the assumption, according to which, the edges of the band cracks have an insignificant imperfections and the degree of this imperfections is characterized through the infinitesimal parameter ε. As in the references [1-5], the sought quantities are presented in power series form with respect to this small parameter:

$$\{\sigma_{ij}^{(k)};\varepsilon_{ij}^{(k)};u_i^{(k)}\} = \sum_{q=0}^{\infty} \varepsilon^q \{\sigma_{ij}^{(k),q};\varepsilon_{ij}^{(k),q};u_i^{(k),q}\}, \varepsilon << 1. \tag{6}$$

Substituting the expression (5) into the Eqs. (2)-(4) and doing some lengthly mathematical manupulations, solution to the (1)-(4) nonlinear boundary value problem is reduced to the solutions of the series linear boundary value problems *qth* of which is obtained by grouping of coefficient of the power ε^q. The problems obtained for the cases where *q=0,1,2,…* are named as the zeroths, first, second…. approximations, respectively. As in the references [1-5] it is established that for the buckling delamination problems it is enough to use only the zeroth and the first approximations in the foregoing series, therefore here we also use only the zeroth and first approximations for determination of the critical values of the external forces. The solution to the zeroth approximation can be obtained analytically within the scope of the certain assumptions [3, 4]. The solution of the boundary value problem related to the first approximation will be obtained numerically with the help of three-dimensional finite element modeling.

According to the FEM modelling, the solution domain is divided into a finite number of finite elements, i.e. the rectangular prism-shaped finite elements and the displacements in the direction of the three axes and the electric potential ϕ are selected as unknowns at each node. Moreover, for the finite element modeling, the following functional Π that expresses the total electro-mechanical energy accumulated in the considered plate [4,6] is used,

$$\Pi(u_1^{(r_n),1}, u_2^{(r_n),1}, u_3^{(r_n),1}, \phi^{(r_n),1}) = \sum_{n=1}^{3} \iiint_{\Omega_n} \left[\frac{1}{2} G_{ijkl}^{(r_n),1} \frac{\partial u_i^{(r_n),1}}{\partial x_j} \frac{\partial u_k^{(r_n),1}}{\partial x_l} \right. \\ \left. + R_{ijk}^{(r_n)} \frac{\partial \phi^{(r_n),1}}{\partial x_i} \frac{\partial u_j^{(r_n),1}}{\partial x_k} - \frac{1}{2} \varepsilon_{ij}^{(r_n)} \frac{\partial \phi^{(r_n),1}}{\partial x_k} \frac{\partial \phi^{(r_n),1}}{\partial x_l} \right] d\Omega_n . \tag{7}$$

With the help of this functional and the known Ritz technique, the solution to the first approximation is reduced to the solution of the corresponding algebraic equations obtained from the first variation of the functional (6). After solution to this system of algebraic equations the values of the critical forces are determined from the initial imperfection criterion detailed in the references [1-5].

3. NUMERICAL RESULTS

Before analyzing of the numerical results obtained from the solution of the problem under consideration, we test the algorithms and programs composed by the authors and used under obtaining of the present results with the known ones given in the literature. For this purpose, we consider the results illustrated in Table I, which show the critical forces for the isotropic rectangular thick plate with band cracks obtained for various NDOF under ($\ell_0/\ell_1=0.5$, $h/\ell_1=0.15$). Note that in this table N_x, N_y and N_z indicate the number of the finite elements along the directions Ox_1, Ox_2 and Ox_3 respectively and according to symmetry properties of the problem, numerical calculations are made in the 1/8 part of the solution domain (Fig. 1b). Note that in Table I it is also given the corresponding ones obtained in the paper [2]. The comparison of the present result with those obtained in the paper [2] illustrate again the trustiness of the present algorithm and PC programs. Moreover, the comparison of the results obtained for various values of the NDOF shows the convergence of the used algorithm.

The mechanical and electro-mechanical properties of the different PZT materials which are used for obtaining the critical forces are presented in Table II. At the same time under obtaining these results two different core materials (Steel (shortly St) and Aluminum (shortly Al)) are used. The modulus of elasticity and Poisson ratio for these materials are taken as $E_{St}=197\times10^9$ Pa, $\nu_{St}=0.2722$, $E_{Al}=70\times10^9$ Pa and $\nu_{Al}=0.3$.

Tables III and IV show the values of the dimensionless critical forces $p_{cr} \big/ C_{44}^{PZT-4}$ and $p_{cr} \big/ C_{44}^{PZT-5H}$ for the PZT-4 and PZT-5H piezoelectric materials, respectively, in the case where the material of the core layer is Al. In these tables the mentioned results are presented for various cracks length *(ℓ₀)* and for various values of the parameter γ $(=\ell_3/\ell_1)$ (Figure 1) in the cases where $e_{ij}=\varepsilon_{ij}=0$ (numerator) and $e_{ij}\neq0$, $\varepsilon_{ij}\neq0$ (denominator). It follows from these numerical results that the values of critical forces with γ close to a certain limit value related to the corresponding plane-strain state and considered in [3].

An important effect in this study is the effect of the interaction between the electrical and the mechanical fields on the critical parameters. This effects can be estimated through the difference of between the critical values obtained in the cases $e_{ij}=\varepsilon_{ij}=0$ (Case 1) and $e_{ij}\neq0$, $\varepsilon_{ij}\neq0$ (Case 2). According to the analyses of the mentioned difference, it is obtained that, the critical forces related to Case 2 are always greater than that obtained in Case 1. Note that in the last column in these tables, the critical forces regarding the stability loss of the whole plate are given.

Table I. $p_{cr} / \mu \left(\mu = \dfrac{E}{2(1+\nu)} \right)$ critical values of buckling delamination force

(N_x, N_y, N_z)	NDOF	h_F		
		0.025	0.0375	0.05
(12,40,40)	13188	0.0124	0.0222	0.0340
(24,40,40)	23772	0.0123	0.0220	0.0339
(24,60,40)	35112	0.0123	0.0219	0.0338
(24,100,40)	57792	0.0122	0.0219	0.0338
(24,100,60)	85312	0.0122	0.0218	0.0337
(24,100,100)	140352	0.0120	0.0217	0.0335
Akbarov vd. (2010)[2]		0.0120	0.0217	0.0336

Table II. The mechanical, piezoelectric and dielectric constants of some piezoelectric materials

Materials (Yang, 2005)	$c_{11}^{(r_1)}$	$c_{12}^{(r_1)}$	$c_{13}^{(r_1)}$	$c_{33}^{(r_1)}$	$c_{44}^{(r_1)}$	$c_{66}^{(r_1)}$	$e_{31}^{(r_1)}$	$e_{33}^{(r_1)}$	$e_{15}^{(r_1)}$	$\varepsilon_{31}^{(r_1)}$	$\varepsilon_{33}^{(r_1)}$
PZT-4	13.9	7.78	7.40	11.5	2.56	3.06	-5.2	15.1	12.7	0.646	0.562
PZT-5H	12.6	7.91	8.39	11.7	2.30	2.35	-6.5	23.3	17.0	1.505	1.302
$BaTiO_3$	15.0	6.53	6.62	14.6	4.39	4.24	-4.3	17.5	11.4	0.987	1.116
	$\times10^{10}$ Pa						C/m^2			$\times10^{-8} C/Vm$	

Table III. The $p_{cr} \Big/ C_{44}^{PZT-4}$ values for different $\gamma\ (= \ell_3 / \ell_1)$ and ℓ_0 / ℓ_1 values of PZT-4/Aluminum/PZT-4 sandwich rectangular thick plate $\left(\dfrac{e_{ij} = \varepsilon_{ij} = 0}{e_{ij} \neq 0, \varepsilon_{ij} \neq 0} \right)$

$\gamma = \ell_3 / \ell_1$	ℓ_0 / ℓ_1						
	0.0	0.2	0.3	0.4	0.5	0.6	0.7
1	$\frac{0.0335}{0.0385}$	$\frac{0.1051}{0.1252}$	$\frac{0.0656}{0.0766}$	$\frac{0.0450}{0.0523}$	$\frac{0.0334}{0.0389}$	$\frac{0.0263}{0.0310}$	$\frac{0.0217}{0.0262}$
3	$\frac{0.0294}{0.0343}$	$\frac{0.1029}{0.1217}$	$\frac{0.0626}{0.0721}$	$\frac{0.0415}{0.0471}$	$\frac{0.0293}{0.0343}$	$\frac{0.0218}{0.0254}$	$\frac{0.0168}{0.0208}$
6	$\frac{0.0267}{0.0312}$	$\frac{0.0955}{0.1114}$	$\frac{0.0589}{0.0686}$	$\frac{0.0382}{0.0446}$	$\frac{0.0270}{0.0315}$	$\frac{0.0194}{0.0229}$	$\frac{0.0144}{0.0179}$
Akbarov, Yahnioglu (2013) [3]	$\frac{0.0260}{0.0300}$	$\frac{0.0900}{0.1100}$	$\frac{0.0575}{0.0660}$	$\frac{0.0375}{0.0430}$	$\frac{0.0260}{0.0300}$	$\frac{0.0190}{0.0220}$	$\frac{0.0140}{0.0170}$

An increase of the critical forces as a result of the piezoelectricity of the face layers of the plate is explained with the "stiffening effect". That is, since a part of the applied compressive force is spent to polarization in the structure of the material, hence the critical value of buckling delamination force increases. Furthermore, as γ and the crack length *(ℓ_0)* increases, the values of the critical external force decrease, and these values approach a certain asymptote as $\gamma \to \infty$.

Table V shows the values of the critical forces obtained also for the PZT-5H/Al/PZT-5H plate under various ℓ_0 and h_F in the case where $h/\ell_1=0.2$ and $\ell_3= \ell_1$ $(\gamma=1)$. As can be seen from this table that the critical forces increase with h_F and decrease with the ℓ_0. Furthermore, the difference between the critical external compressive forces obtained in the cases $e_{ij}\neq 0$, $\varepsilon_{ij}\neq 0$ and $e_{ij}=\varepsilon_{ij}=0$ also increases with the PZT layers thickness (h_F). There exists such a value of ℓ_0/ℓ_1 (denoted by (ℓ_0/ℓ_1)*) after which, i.e. under $(\ell_0/\ell_1)^*/ < \ell_0/\ell_1$ the local buckling delamination around the interface cracks takes place at an earlier stage of the loading than the buckling of the whole plate. But in the cases where $(\ell_0/\ell_1)^*/ > \ell_0/\ell_1$ the buckling of the whole plate takes place earlier than the local buckling delamination around the interface cracks. According to the author's point of view, critical buckling force is closely related the ratio between the length and the thickness of the buckling part. Hence, for small size of the crack length, if the ratio between the length of the crack and the thickness of the PZT layer is greater than the ratio of corresponding size of the whole plate then critical buckling force of the whole plate is less than that for the buckling delamination of this plate.

Table IV. The $p_{cr} \big/ C_{44}^{PZT-5H}$ values of the PZT-5H/Aluminum/PZT-5H sandwich rectangular thick plate for different $\gamma\,(=\ell_3/\ell_1)$ and ℓ_0/ℓ_1 values $\left(\dfrac{e_{ij}=\varepsilon_{ij}=0}{e_{ij}\neq 0, \varepsilon_{ij}\neq 0}\right)$

$\gamma = \ell_3/\ell_1$	ℓ_0/ℓ_1				
	0.2	0.3	0.4	0.5	0.7
1	$\frac{0.3301}{0.4098}$	$\frac{0.2000}{0.2502}$	$\frac{0.1345}{0.1708}$	$\frac{0.0982}{0.1274}$	$\frac{0.0628}{0.0868}$
3	$\frac{0.3240}{0.3981}$	$\frac{0.1918}{0.2347}$	$\frac{0.1249}{0.1527}$	$\frac{0.0872}{0.1068}$	$\frac{0.0493}{0.0607}$
5	$\frac{0.3235}{0.3971}$	$\frac{0.1904}{0.2336}$	$\frac{0.1235}{0.1514}$	$\frac{0.0858}{0.1054}$	$\frac{0.0479}{0.0593}$
7	$\frac{0.3232}{0.3968}$	$\frac{0.1899}{0.2332}$	$\frac{0.1230}{0.1509}$	$\frac{0.0853}{0.1049}$	$\frac{0.0474}{0.0588}$
9	$\frac{0.3231}{0.3965}$	$\frac{0.1896}{0.2329}$	$\frac{0.1227}{0.1506}$	$\frac{0.0850}{0.1046}$	$\frac{0.0471}{0.0585}$

Table V. The $p_{cr} \big/ C_{44}^{PZT-5H}$ values of the PZT-5H/Aluminum/PZT-5H sandwich rectangular thick plate for different h_F and ℓ_0 values $\left(\dfrac{e_{ij}=\varepsilon_{ij}=0}{e_{ij}\neq 0, \varepsilon_{ij}\neq 0}\right)$

h_F/ℓ_1	ℓ_0/ℓ_1							
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7
0.0166	$\frac{0.1280}{0.1820}$	$\frac{0.3694}{0.3758}$	$\frac{0.1154}{0.1213}$	$\frac{0.0592}{0.0658}$	$\frac{0.0351}{0.0354}$	$\frac{0.0239}{0.0242}$	$\frac{0.0176}{0.0182}$	$\frac{0.0140}{0.0160}$
0.0333	$\frac{0.1321}{0.1856}$	$\frac{0.5203}{0.5946}$	$\frac{0.2286}{0.2604}$	$\frac{0.1246}{0.1432}$	$\frac{0.0790}{0.0926}$	$\frac{0.0556}{0.0672}$	$\frac{0.0422}{0.0529}$	$\frac{0.0339}{0.0451}$
0.0500	$\frac{0.1398}{0.1902}$	$\frac{0.6097}{0.7401}$	$\frac{0.3301}{0.4098}$	$\frac{0.2000}{0.2502}$	$\frac{0.1345}{0.1708}$	$\frac{0.0982}{0.1274}$	$\frac{0.0765}{0.1023}$	$\frac{0.0628}{0.0868}$
0.0666	$\frac{0.1440}{0.1960}$	$\frac{0.6729}{0.8701}$	$\frac{0.4099}{0.5441}$	$\frac{0.2703}{0.3606}$	$\frac{0.1920}{0.2582}$	$\frac{0.1453}{0.1979}$	$\frac{0.1160}{0.1611}$	$\frac{0.0972}{0.1380}$

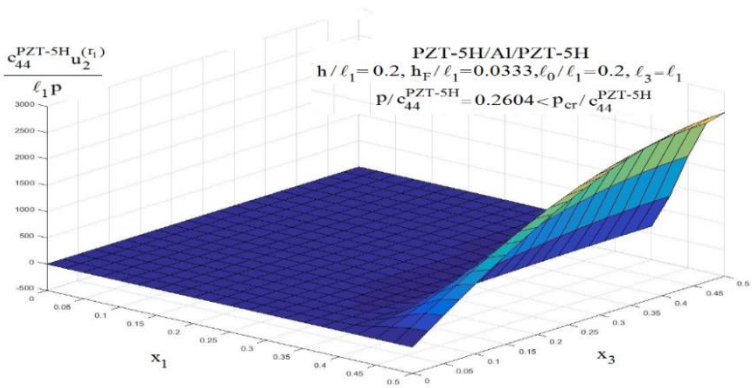

Figure 2. The surface graph of the PZT-5H/Aluminum/PZT-5H sandwich rectangular thick plate for $p/c_{44}^{PZT-5H}=0.2604$ in the section $x_2=h_F+h_C+0^+$

In Figure 2, the crack edge surface displacement graph is given for the PZT-5H/Al/PZT-5H plate in the case where $h/\ell_1=0.2$; $h_F/\ell_1=0.0333$; $\ell_0/\ell_1=0.2$ and $\ell_3=\ell_1$. Accordingly, as the external compressive force approaches to its critical value, the values of the displacements of the surface increase and grow very much. It is also seen that as the external pressure force increases, the crack surface form is in agreement with the form of the initial imperfection of the cracks' edge surface.

In Figure 3 and Figure 4, the numerical results obtained for different PZT face layers materials in cases where the core layer is Al (Figure 3) and St (Figure 4) are given. In these figures straight (dashed) lines are obtained in the case where $e_{ij}\neq 0,\ \varepsilon_{ij}\neq 0$ (in the case where $e_{ij}=\varepsilon_{ij}=0$). Also these figures show that the following relationship is provided for the critical values obtained for different PZT materials:

$$p_{cr}/c_{44}^{PZT-5H} > p_{cr}/c_{44}^{PZT-4} > p_{cr}/c_{44}^{BaTiO_3}. \tag{8}$$

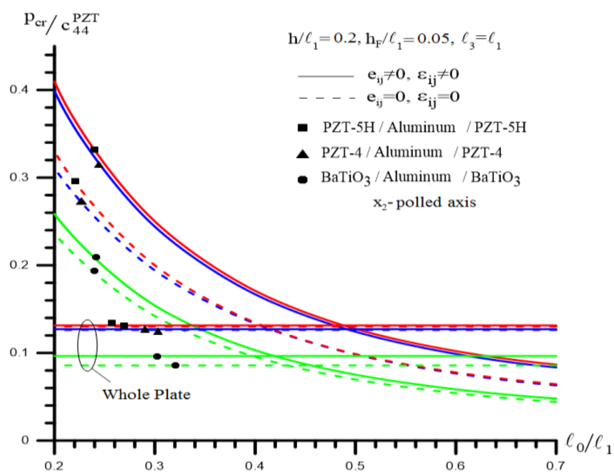

Figure 3. The critical buckling delamination force values of the PZT/Aluminum/PZT sandwich rectangular thick plate for the different PZT materials and ℓ_0 crack lengths

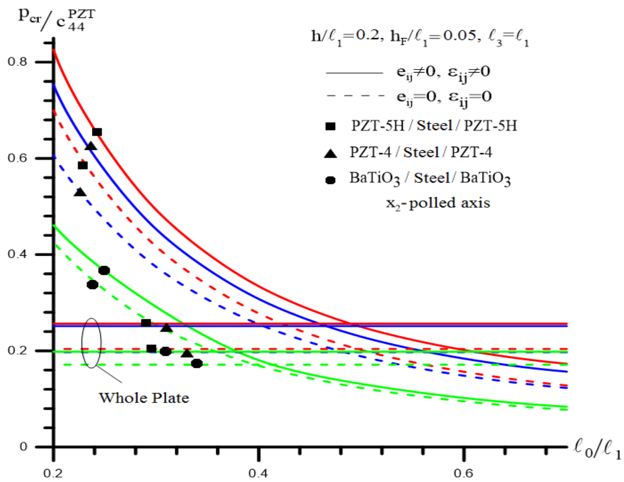

Figure 4. The critical buckling delamination force values of the PZT/Steel/PZT sandwich rectangular thick plate for the different PZT materials and ℓ_0 crack lengths

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