APPLICATION OF A NOVEL THERMO-ECOLOGICAL PERFORMANCE CRITERION: EFFECTIVE ECOLOGICAL POWER DENSITY (EFECPOD) TO A JOULE-BRAYTON CYCLE (JBC) TURBINE

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ABSTRACT
This study presents an application of a new performance analysis criterion named as Effective Ecological Power Density (EFECPOD) to a Joule-Brayton cycle (JBC) turbine. The turbine performance is expressed a single value by the proposed criterion using effective efficiency, effective power, cycle temperature ratio and volume. NOx formation and turbine dimensions are considered by the cycle temperature ratio and turbine volume, respectively. The turbine volume is also related to production cost of the heat engine. Therefore, the proposed criterion is essential for multi purpose optimization. Furthermore, this criterion can be developed and applied to the other gas cycle and heat engines. Also, the influences of engine design parameters such as cycle temperature ratio, pressure ratio, turbine speed, and equivalence ratio on the EFECPOD have been examined based on Finite-Time Thermodynamics Modelling (FTTM). In order to obtain realistic results, temperature-dependent specific heats for working fluid have been used and heat transfer and exhaust output losses have been taken into consideration. The results presented could be an essential tool for JBC turbine designers.

Keywords: Joule-Brayton Cycle, Thermodynamic Analysis, Performance Optimization, Gas Turbine

INTRODUCTION
Gas turbines are used in so many places such as ships, tanks, air planes, power plants etc. in order to provide mechanical energy and electrical energy. In the literature so many studies have been done about the gas turbines and their cycle named as Joule-Brayton cycle.

Zare et al. [1] conducted an exergoeconomic investigation on a combined cycle in which the waste heat from the Gas Turbine-Modular Helium Reactor (GT-MHR) is recovered by an ammonia–water power/cooling cogeneration system using parametric analyses. They claimed that the unit cost of products was decreased by 5.4% by combining these two cycles. However, total investment cost rate increased just about 1%. Singh et al. [2] proposed a performance scheme for Extremum-seeking control of a supercritical carbon-dioxide closed Brayton cycle coupled with solar thermal power plant. Singh et al. [3] examined the Dynamic characteristics of a direct-heated supercritical closed Brayton power conversion system with carbon-dioxide in a solar thermal power plant. Abd El-Maksoud [4] increased the performance of a gas turbine by combining isothermal concept and binary Brayton cycle. They proposed this combination for future power generation. Singh et al. [5] examined the effect of the relative volume-ratios on dynamic characteristics of a closed Brayton cycle with supercritical carbon-dioxide in a solar-thermal power plant. Ferraro et al. [6] thermodynamically analyzed the performance of solar plants running with cylindrical parabolic collectors and air turbine engines in an open Joule–Brayton cycle using an improved model. Haseli [7] carried out an optimization study for a regenerative Brayton cycle by maximization of second law efficiency. Plaznik et al. [8] theoretically and experimentally analyzed the effects of different magnetic thermodynamic cycles such as Brayton, Ericsson and Hybrid Brayton–Ericsson cycles on the performance of a magnetic cooling device with an active magnetic regenerator. Malinowski and Lewandowska [9] presented a universal analytical model based on the Brayton cycle for part-load operation of gas microturbines to analyze energetic and exergetic efficiencies. Sim et al. [10] developed a micro-power pack using automotive alternators powered by a micro-gas turbine based on simple Brayton cycle to recharge battery packs of electric vehicles. Rao and Francuz [11] provided an identification and assessment for advanced performance improvements of combined cycles in coal based power systems. Ablay [12] proposed a new modeling approach and load following control strategy for gas turbine nuclear power plants so as to assess a way for concept designs. They presented computational results to prove the validity and effectiveness of the proposed...

This study reports a new performance analysis criterion named as EFECPOD includes the effective efficiency, effective power, cycle temperature ratio and turbine volume. In the literature, there is not any criterion covers effective efficiency, effective power, maximum combustion temperatures and turbine dimensions altogether, based on finite time thermodynamics. Furthermore, a comprehensive comparison for the design parameters such as pressure ratio, turbine speed, turbine diameter, turbine length, heat transfer coefficient, equivalence ratio has been presented. The results could be used by real turbine designers to optimize the JBC gas turbines in terms of dimensions, performance and NOx emissions.

**THEORETICAL MODEL**

![P-v diagram for the irreversible Diesel-Miller cycle](image)

This study presents a comprehensive analysis for JBC which is depicted in Figure 1. A numerical simulation of EFECPOD is carried out based on FTTM. In the analysis, pressure ratio \( \lambda \), residual gas fraction (RGF), turbine speed \( N \), inlet temperature \( T_i \), inlet pressure \( P_i \), combustion chamber wall temperature \( T_W \), mass flow rate of the air \( m \), heat transfer coefficient \( h_{tr} \), turbine bore \( b \) and length \( L \) are defined as follow: 4, 5%, 3000 rpm, 300 K, 100 kPa, 400 K, 1 kW/m2K, 0.5 m, 1 respectively, at the standard conditions.

In the present model, the evaluated criteria called effective ecological power density, effective power and effective efficiency could be stated as follow:

\[
EFECPOD = \frac{\eta_{ef} P_{ef}}{\eta_{out}} = \frac{\dot{Q}_{in} - \dot{Q}_{out}}{P_{ef}}, \frac{\eta_{ef}}{\eta_{out}} = \frac{P_{ef}}{\dot{Q}_{out}}
\]

Where, the total heat addition \( \dot{Q}_{in} \) at constant pressure (2-3) and the total heat rejection \( \dot{Q}_{out} \) at constant volume (4-1) could be written as below:

\[
\dot{Q}_{in} = \dot{Q}_{f} - \dot{Q}_{hd} = m_{ef} \int_{T_f}^{T_i} C_{p} dT
\]
\[ Q_{out} = \dot{m}_T \left[ T_f C_p dT \right] \] (3)

Where \( T_p \) and \( T_f \) are the ambient temperature and the temperature at the beginning of the compression. \( \alpha \) is cycle temperature ratio and it is stated as below:

\[ \alpha = \frac{T_{max}}{T_{min}} = \frac{T_s}{T_f} \] (4)

Where \( \eta_c \) is isentropic efficiency for the compression process, \( \dot{\lambda} \) is pressure ratio and it may be expressed as:

\[ \dot{\lambda} = P_2 / P_1 \] (5)

Where \( Q_f \) is the total heat potential of the injected fuel and it is given as below:

\[ \dot{Q}_f = \dot{m}_f H_u \] (6)

Where \( H_u \) is lower heat value (LHV). \( \dot{m}_f \) is time-dependent fuel mass and it can be expressed as follows:

\[ \dot{m}_f = \frac{m_f N}{60} \] (7)

Where \( \dot{m}_f \) is fuel mass per cycle (kg). \( \dot{Q}_{f,c} \) is heat released by combustion; \( \dot{Q}_{hu} \) is the heat loss by heat transfer into cylinder wall and they are given as below:

\[ \dot{Q}_{f,c} = \eta_c \dot{m}_f H_u \] (8)

\[ \dot{Q}_{hu} = h_u A_{sur} (T_{wall} - T_w) = h_u A_{sur} \left( \frac{T_s + T_f}{2} - T_w \right) \] (9)

Where, \( \eta_c \) is combustion efficiency. It can be written as below [21-24]:

\[ \eta_c = -1,44738 + 4,18581 / \phi - 1,86876 / \phi^2 \] (10)

\( \phi \) is equivalence ratio and it can be written as below:

\[ \phi = \frac{m_f / m_a}{F_u} \] (11)

Where, \( m_a \) is air mass per cycle (kg), \( V_T \) is total turbine volume, \( A_{sur} \) is surface area where heat transfer is carried out, \( F_u \) is stoichiometric fuel-air ratio [25] and they are given as follow:

\[ m_a = \rho_a V_u = \rho_a (V_T - V_{rg}) \] (12)

\[ V_T = V_{max} = m_r \nu_4 = \frac{\pi b^2 L}{4} \] (13)

\[ A_{sur} = \frac{4V_T}{b} \] (14)

\[ F_u = \frac{28.85}{(12.01 \cdot \alpha + 1.008 \cdot \beta + 16 \cdot \gamma + 14.01 \cdot \delta)} \] (15)

Where \( b \) and \( L \) are turbine bore (inner diameter) and turbine length, \( \nu_4 \) is specific volume of the state point 4 in which the specific volume of cycle is the maximum. \( m_f \) is the total mass of the working fluid and it can be expresses as follows:

\[ m_f = m_a + \dot{m}_f + \dot{m}_{rg} \] (16)

Where \( \dot{m}_{rg} \) is mass residual gas (kg) per cycle, which is given as:

\[ \dot{m}_{rg} = \rho_{rg} V_{rg} \] (17)

Where \( \rho_a \) and \( \rho_{rg} \) are the densities of the air and residual gas in the turbine, they can be obtained from functions below:

\[ \rho_a = f(T_r, P_r) \] (18)
\[ \rho_{rg} = f(T_{mix}, P) \]  

Where \( T_{mix} \) is average temperature of air-steam mixture. They are given as below:

\[ T_{mix} = \frac{m_{1}TR_{1} + m_{2}TR_{rg}}{m_{1}R_{1} + m_{2}R_{rg}} \]  

\( R_{a} \) and \( R_{rg} \) are gas constants of air and residual gas. Their values are taken as 0.287 kJ/kg.K

The compression ratio \( (r) \) is given as:

\[ r = V_{1} / V_{2} \]  

Time-dependent values of \( m_{a} \) and \( m_{rg} \) can be attained as below:

\[ m_{a} = \frac{m_{a}N}{60} = \frac{m_{a}F_{r}}{\phi} \]  

\[ m_{rg} = \frac{m_{rg}N}{60} = m_{r} \text{RGF} \]  

Where \( N \) and RGF are the turbine speed and the residual gas fraction, \( V_{a} \) and \( V_{rg} \) are volumes of air and residual gas, \( f \) stands for function. The functional expressions are obtained by using EES software [26] Where subscript "1" stands for the condition before the compression process (state point 1). \( T_{1} \) and \( P_{1} \) are in-cylinder temperature and pressure at the beginning of compression process. The fuel used in the model is octane and its chemical formula is given as \( \text{C}_{8}\text{H}_{18} \) [25].

Where \( \alpha, \beta, \gamma, \delta \) are atomic numbers of carbon, hydrogen, oxygen, nitrogen in fuel, respectively. \( \varepsilon \) is molar fuel-air ratio [25]:

\[ \varepsilon = \frac{0.21}{\left( \alpha - \frac{\gamma}{2} + \frac{\beta}{4} \right)} \]  

Where \( T_{me} \) and \( T_{w} \) are mean combustion temperature and cylinder wall temperature. \( C_{p} \) and \( C_{v} \) are constant pressure and constant volume specific heats, they could be written for the temperature range of 300-3500 K as below [27]:

\[ C_{p} = 2.506 \times 10^{-11}T^{2} + 1.454 \times 10^{-7}T^{1.5} - 4.246 \times 10^{-7}T + 3.162 \times 10^{-5}T^{0.5} + 1.3301 \times 10^{-4}T^{2} + 3.063 \times 10^{-2}T - 2.212 \times 10^{-3}T^{3} \]  

\[ C_{v} = C_{p} - R \]  

The equations for reversible adiabatic processes (1-2s) and (3-4s) are respectively as follows:

\[ C_{v1} \cdot \ln \left| \frac{T_{2s}}{T_{1}} \right| = R \ln |r| \cdot C_{v1} \cdot \ln \left| \frac{T_{4s}}{T_{3}} \right| = R \ln |r| \]  

Where,

\[ C_{v1} = 2.506 \times 10^{-11}T_{2s1}^{2} + 1.454 \times 10^{-7}T_{2s1}^{1.5} - 4.246 \times 10^{-7}T_{2s1} + 3.162 \times 10^{-5}T_{2s1}^{0.5} + 1.0433 - 1.512 \times 10^{-4}T_{2s1}^{1.5} + 3.063 \times 10^{-2}T_{2s1} - 2.212 \times 10^{-3}T_{2s1}^{3} \]  

\[ C_{v2} = 2.506 \times 10^{-11}T_{4s3}^{2} + 1.454 \times 10^{-7}T_{4s3}^{1.5} - 4.246 \times 10^{-7}T_{4s3} + 3.162 \times 10^{-5}T_{4s3}^{0.5} + 1.0433 - 1.512 \times 10^{-4}T_{4s3}^{1.5} + 3.063 \times 10^{-2}T_{4s3} - 2.212 \times 10^{-3}T_{4s3}^{3} \]  

\[ T_{2s1} = \frac{T_{2s1} - T_{1}}{\ln \frac{T_{2s1}}{T_{1}}} \cdot \frac{T_{4s3} - T_{3}}{\ln \frac{T_{4s3}}{T_{3}}} \]  

\[ \beta = \frac{P_{3}}{P_{2}} = \frac{T_{3}}{T_{2}} \]  

\( \beta \) is named as pressure ratio. For irreversible conditions, \( T_{2s} \) and \( T_{4s} \) could be written as below:

\[ T_{2} = T_{2s} + T_{1} \left( \eta_{c} - 1 \right) \]  

\[ T_{4} = T_{4s} + \eta_{e} \left( T_{4s} - T_{1} \right) \]  

Where \( \eta_{c} \) and \( \eta_{e} \) are isentropic efficiencies for the compression and expansion processes, respectively.
RESULTS AND DISCUSSION

In this study, a new performance analysis criterion named as EFECPOD and FTTM have been applied to a JBC gas turbine. Figure 2 show the effects of pressure ratio on the EFECPOD with respect to equivalence ratio. There are optimum points of the equivalence ratio for the EFECPOD at the constant turbine speed and air mass flow rate. The equivalence ratios give the maximum EFECPOD and effective efficiency increase to a particular value and then start to decrease with increasing equivalence ratio. The maximum values of EE and EFECPOD increase with increasing pressure ratio owing to high temperatures and pressures in the combustion chamber. Also, the turbine volume decreases with increasing pressure ratio to provide constant air mass flow rate and turbine speed. Therefore, higher performance can be obtained at the lower turbine volume.

Figures 3 and 4 show the influence of turbine speed on the EE and EFECPOD for constant pressure ratio-air mass per cycle and constant pressure ratio-mass flow rate of the air introduced into the compressor. Total intake air and injected fuel per second enhance with increasing engine speed, therefore, turbine performance increases at the constant cycle air mass conditions. At this condition, the EE and EFECPOD increase since the turbine volume and maximum combustion temperatures increase with increasing turbine speed. However, the turbine volume which is needed to provide constant air mass flow rate abates with increasing turbine speed.

![Figure 2](image1)

**Figure 2.** The variation of EFECPOD with respect to $\eta_{ef}$ at constant $N$ and $\dot{m}$ for different pressure ratios

![Figure 3](image2)

**Figure 3.** The variation of EFECPOD with respect $\eta_{ef}$ at constant $\lambda$ and $m_{air}$ for different turbine speed

Figures 5-6 illustrate the effects of turbine diameter and length on the turbine performance at constant mass flow rate and pressure ratio. While the increase of turbine diameter provides performance increment between 0.5-2.5 m, increase of turbine length leads to a reduction in the performance parameters between 1-5 m. Although both
of the length and diameter are related to turbine dimensions, their changes affect the performance reversely, since the heat transfer area decreases with the diameter increment and the length reduction. Increasing heat transfer area causes to performance retrogression as higher heat transfer loss is occurred.

**Figure 4.** The variation of \( \text{EFECPOD} \) with respect to \( \eta_{ef} \) at constant \( \lambda \) and \( \dot{m} \) for different turbine speed

**Figure 5.** The variation of \( \text{EFECPOD} \) with respect to \( \eta_{ef} \) at constant \( \lambda \) and \( \dot{m} \) for different turbine diameter

**Figure 6.** The variation of \( \text{EFECPOD} \) with respect to \( \eta_{ef} \) at constant \( \lambda \) and \( \dot{m} \) for different turbine length
Figure 7. The variation of the EFECPOD with respect to $\eta_{ef}$ at constant $\lambda$ and $\dot{m}$ for different heat transfer coefficients.

Figure 8. The variation of the EFECPOD with respect to $N$ at constant $\lambda$ and $m_{air}$ for different $\phi$, b) $P_{cf}$ - $P_d$. EFECPOD with respect to $N$ at constant $\phi$ and $m_{air}$ for different $\lambda$.

Figure 7 demonstrates the effects of heat transfer coefficient on the EE and EFECPOD at constant mass flow rate and pressure ratio. As expected, the turbine performance abates with increasing heat transfer coefficient since total heat transfer loss raises.

Figure 8 demonstrates the effects of pressure ratio, equivalence ratio and turbine speed together. It is clear that turbine speed and pressure ratio positively affect the performance parameters. However, there is an optimum point for the equivalence ratio which gives the maximum EFECPOD values. It is approximately 0.8 for the EFECPOD. The turbine performance decreases at higher values of equivalence ratio due to lower combustion.
Figure 9. The variation of the EFECPOD with respect to $\lambda$ at constant $N$ and $m$ for different $\phi$

efficiency and higher heat transfer and exhaust losses. As can be seen in the figures, the turbine volume is considerably affected by equivalence ratio and pressure ratio. The turbine volume increases with increasing turbine speed and equivalence ratio; decreasing pressure ratio. The maximum turbine volumes are obtained with 4 of the pressure ratio and 1.1 of the equivalence ratio. At the different $\lambda$ conditions, the maximum combustion temperatures increase and the turbine volumes decreases with increasing pressure ratio. The EFECPOD increases with increasing turbine speed because intake air mass enhances with increasing turbine speed. Another substantial point is that NOx formation is very sensitive the combustion temperatures, thus, it can be said that NOx increases with increasing pressure ratio and turbine speed at the constant $\phi$ and $m_{air}$ conditions.

Figure 9 demonstrates the effects of equivalence ratio on the turbine performance with respect to changing pressure ratio at constant turbine speed and the mass flow rate of the air. It is obvious that the EFECPOD raise with the enhancing pressure ratio. As similar to previous figures, the optimum points of the equivalence ratios which give the maximum EFECPOD are observed. The maximum value of the EFECPOD increases up to a particular value of the equivalence ratio which is 0.8 and then start to decrease. However, the maximum volume is seen at 1.1 of the equivalence ratio.

CONCLUSION
A new performance analysis criterion has been developed and applied to a gas turbine cycle. The effects of the engine design and operating parameters on the performance parameters and energy losses of a gas turbine have been investigated by using the presented analysis criterion. A comprehensive parametrical study has been performed based on numerical examples. In the parametrical studies, the effects of pressure ratio ($\lambda$), turbine speed ($N$), turbine diameter ($b$), turbine length ($L$), equivalence ratio ($\phi$) and heat transfer coefficient ($h_{tr}$) on the performance have been examined. The results showed that the determined performance parameter called effective ecological power density (EFECPOD) increases with pressure ratio, turbine speed, turbine diameter; decrease with turbine height ($L$), heat transfer coefficient ($h_{tr}$). The EFECPOD increases up to a particular value and then begin to decrease with increasing equivalence ratio. The results are scientifically valuable and therefore, they can be assessed by JBC turbine designers.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$A$</td>
<td>heat transfer area (m$^2$)</td>
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<td>$b$</td>
<td>bore (m)</td>
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<tr>
<td>$C_v$</td>
<td>constant volume specific heat (kJ/kg.K)</td>
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<tr>
<td>$C_p$</td>
<td>constant pressure specific heat (kJ/kg.K)</td>
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<td>$F$</td>
<td>fuel-air ratio</td>
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<tr>
<td>FTT</td>
<td>finite-time thermodynamics</td>
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<td>$h_{tr}$</td>
<td>heat transfer coefficient (W/ m$^2$K)</td>
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<td>$H_u$</td>
<td>lower heat value of the fuel (kJ/kg)</td>
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<td>ICE</td>
<td>Internal combustion engines</td>
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<td>Symbol</td>
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<td>temperature (K)</td>
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<td>specific volume (m3/kg)</td>
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**Greek letters**

- \( \alpha \) cycle temperature ratio, atomic number of carbon
- \( \beta \) pressure ratio, atomic number of hydrogen
- \( \delta \) atomic number of nitrogen
- \( \phi \) equivalence ratio
- \( \gamma \) atomic number of oxygen
- \( \lambda \) cycle pressure ratio
- \( \psi \) cut-off ratio
- \( \rho \) density (kg/m3)
- \( \eta_C \) Isentropic efficiency of compression
- \( \eta_E \) Isentropic efficiency of expansion

**Subscripts**

- \( l \) at the beginning of the compression process
- \( a \) air
- \( c \) combustion, clearance
- \( cyl \) cylinder
- \( ef \) effective
- \( f \) fuel
- \( ht \) heat transfer
- \( i \) initial condition
- \( ic \) incomplete combustion
- \( in \) input
- \( l \) loss
- \( max \) maximum
- \( me \) mean
- \( min \) minimum
- \( mix \) mixture
- \( out \) output
- \( rg \) residual gas
- \( s \) isentropic condition
sur  surface
st  stoichiometric
t  total
w  cylinder walls

REFERENCES