Bertrand Competition with Network Effects and Switching Costs: An Agent-based Computational Approach

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Abstract

Economics is a complex system that requires different approaches for analysis. I analyzed switching costs and network effects together with a new simulation approach to modeling systems composed of autonomous and interacting agents. Agent based computational economics is one of the simulation tools and an efficient tool box for complex economic systems. I developed an agent based computational model of duopolistic competition to analyze how the network effects and switching costs shape competitive outcomes by simulation methods.

Keywords: Agent-based Computational Economics, Switching Costs, Network Effects, Simulation

1 Introduction

Switching costs [1] and network effects [2] not only play a major role in high-tech industries but also play a fundamental role in shaping business strategies in the high tech producing industries (Shapiro and Varian, 1998). In many parts of modern economies; competition is increasingly characterized by switching costs and network effects phenomenon of incompatible products. Switching costs and network effects bind customers to vendors if products are incompatible in high-tech industries such as hardware-software industries, telecommunication industries etc. [3].

Separate purchases are the main characteristic for high-tech assets. Both switching costs and proprietary network effects arise when consumers value forms of compatibility that require otherwise separate purchases to be made from the same firm (Farrell & Klemperer, 2007, p.1971). To obtain best market outcome, customers should coordinate their expectations for each separate purchase period. Producers also take into account these essential features of the market to generate present and future price strategies to maximize their profit or customer base. Due to the possibility of shifting market outcome towards low-level equilibrium, both switching costs and network effects have attracted concerns in competition policy regarding its effectiveness [4].

The value which comes from consuming the good received by consumers can be separated into two distinct parts. The first component, labeled as the autarky value, is the value generated by the product even if there are no other users. This autarky value contains both switching cost and individual utility. The second component, which is called synchronization value, is the additional value derived from being able to interact with other users of the product. It is the latter value represents the essence of network effects. Both of these parts exist in a costumer's utility function if there are no externalities regarding network (Liebowitz and Margolis, 1998, p. 1). In spite of this utility functional form, there are a few works concerning both switching cost and network effect (see, e.g., Chen & Forman, 2006, Farrell & Klemperer ,2007, Maicas & Polo & Sese, 2009, Suleymanova

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[1]Switching costs are the real or perceived costs of changing to another firm's products, even when those products may be functionally identical. There can be lot of different forms of switching cost like transaction cost, learning cost etc. Switching costs create various incentives for firms to change their price decisions (see OFT, 2003).

[2]Utility of product which user derives from consumption of the good increases with the number of other agents consuming the good (see Katz & Shapiro, 1985)

[3]The empirical literature on switching costs and network effect is situated in many other areas such as computer software, supermarkets , air travel, alliances of airlines in different frequent-flyer programs, phone services, television, electricity, automobile insurance, telecommunications, video recording etc (see Farrell & Klemperer, 2007).

[4]This competition effectiveness is not just about maximizing mutual utility. Antitrust policy, innovation issues, intellectual property rights, international policy issues are all key policy aspects of network effect and also switching cost (see Gandal, 2002).

& Wey, 2011, Doganoglu & Grzybowski, 2013, Weiergräber, 2014, Chen, 2015/a, 2015/b). Most literature is focused entirely on either one of the two cases exclusively.

Klemperer (1987a/b) shows "bargain and rip off" structure [5] in a two-period market environment with consumer switching costs. This pricing strategy expresses "low to attract business, high to extract surplus". A firm with a larger customer base puts relatively more weight on harvesting this base than on winning new customers (fat-cat effect): "Large shares tend to shrink and small shares tend to grow" (Farrell & Klemperer, 2007, p.1974). This behavior changes market shares (Beggs and Klemperer, 1992). However, Klemperer and his followers have usually used this structure for switching cost, synchronization value creates same incentives for firms (Farrell and Shapiro, 1988). Firms deal with the trade-off issue between harvesting and investing which are the main strategies for firms in multiple-period models. The main goal of investing strategy is to get customers locked into relevant technology or goods. Locking-in customers or even markets in to early choices makes it possible to obtain extra profit from locked-in customers in later periods. Extra profit within current periods is possible with harvesting strategy.

As with switching cost literature, network effects literature has also become very popular. These works usually focused on markets in regards to consumer adaptation decisions and to the result of these decisions. Adaptation can occur sequentially or simultaneously and customer lock-in typically leads to a monopolization outcome as well as several dynamic inefficiencies (see, e.g. Farrell and Saloner, 1986, Katz and Shapiro, 1986, 1994, Arthur, 1989, Mitchell and Skrzypacz, 2006). From a cooperative game theory perspective, coordination failures are one of the most controversial topics in network effect literature (Suleymanova, 2010, p.5-6). If the size of the coalition increases, buyer's surplus will increase (and vice versa). But the coordination on most effective outcome is not an easy issue that some communication devices, initial adoptions and expectations form the coordination level. These conditions affect the performance of competition among networks.

Both switching costs and network effects literature contains analytical and empirical analyses which always use general assumptions that are unable to encapsulate entire real world issues like other economic topics. So the complexity surrounding our environment requires us to seek new methodologies and disciplines. Our environment cannot be clarified by only a singular discipline. It also requires multiple disciplines analysing relevant questions. Looking from the viewpoint of economics, computer science plays a key role in exploring real world issues which should be studied without unrealistic assumptions. Conventional economic theory, following the style of mathematics in general and real analysis in particular, begins with a set of definitions and assumptions (Judd, 2006). This ensures the inclusion of the environmental aspect and a model less complicated. Complex structures with their interacting parts cannot be predicted easily therefore classical techniques which usually prefer a reductionist approach become ineffective. The reductionist approach represents the system equal to the sum of its components. Even if the components are not complicated, their interactions transform the main structure into complexity. For example in network effect cases, even though coordination cases are analyzed, interactions between adopters, which is the key term for emergence property, are usually ignored.

A model is a prototype that describes real world structures. Modelling includes the process of mapping the problem from the real world to its model. To handle complicated issues in models, new approaches should be considered. The newly developing field of agent-based computational economics (ACE) is defined as the computational study (simulation) of economies modelled as evolving systems of autonomous, interacting agents (Tesfatsion, 2000, p.1-4). ACE begins with initial agent conditions and their interactions but these conditions do not like assumptions generally used by analytical models. These conditions are made more flexible in order to test model with different situations. This enormous testing environments requires computer systems to simulate these different conditions which generate dynamic consequences. Distinguishing between analytical and simulation models may be useful since analytical ones can often be insufficient for complex systems.

Borshchev and Filippov (2004, p.1) explain simulation model as set of rules which define model characteristics and how it will change during the simulation , given its present state. These rules can be equations, flowcharts, state machines, cellular automata and agent based rules. A simulation is the process of model execution that takes the model through state changes over time. For complex problems, simulation modelling is a better approach than conventional ones. This approach can be useful for analyzing together both switching costs and network effect issues. A simulation is useful for understanding how macro scale effects arise from the micro processes of interactions among many agents. So network effect can be simulated not just like a micro

[5] Switching costs impact on the structure of prices on multiperiod repeated purchases that allow firms to price above cost to consumers once they have purchased the product and are locked-in, as the consumer would incur a cost to changing supplier. These customers become extremely valuable for firms. As a result, competition can mean that firms price very low, even below cost to attract new customers (see OFT, 2003. part 1.3).

process, but also a macro one that shows feedback structure. A simulation does not prove theorems directly as in deduction but as deduction starting with a set of explicit assumptions (conditions). A simulation is also suitable for induction by generating its own data dynamically. Simulation differs from deduction and induction process in both its implementation and goals and provides understanding of systems through controlled computational experiments (Axelrod & Tesfatsion, 2005, p.3-4).

Tesfatsion (2005, p.4) expresses local interactions as a major role in giving rise to global patterns. Large numbers of micro local agents interact repeatedly (simultaneously or sequentially) so that these interactions cause global regularities which feed back into the determination of local interactions. The result is an intricate and complex system of interdependent feedback loops connecting micro behaviours, interaction patterns, and global regularities. Economies are complex dynamic systems, which is why complex modelling concepts are being discussed in economic literature. Mainstream economic models are inadequate in embedding real world facts. First of all this deficiency must be overcome by new toolboxes. In this regard agent based modelling is a new approach to simulating complex systems composed of cognitive, heterogeneous, interacting, autonomous agents which are powerful candidates for dealing with real world issues. These agents may be consumers, sellers, firms, banks, social groups, political groups, investors and policy makers. Social science is not only composed of individual agents but also interactions that are created by these individuals. Interactions enable models to have an opportunity to analyze crowding effects. Unexpected situations arise from this crowding effect which cannot be modelled, programmed or predicted with an agent's own properties. There is no oppurtunity to properly code this effect explicitly. Consequently, agent based modelling offers a way to model social systems that are composed of agents which interact and influence each other and learn from environmental, interactions and experience and that provide an opportunity to adopt their behaviors (Macal & North, 2010). This modelling concept is based on bottom-up simulation rather than top-down macro decision-making. Behavior at the individual level (buttom) generates higher level structures (up) which feed back to the lower level.

In this paper I have developed an agent based computational model of duopolistic competition to analyze how the network effects and switching costs shapes competitive outcomes by simulation methods I have attempted to express above. In this model both network effects and switching costs are essential features of the market. This paper differs dramatically from main stream literature in that it does not make strict assumptions of agent homogeneity [6], learning, rationality [7] and global network effect [8]. The customer's utility function has a "distance" variable that is randomly set for every customer which has an adaptive learning path and a different neighborhood structure that is generated randomly. All these functional structure make customers be heterogeneous. Customers attach importance to their habits which switching cost value increases with. This situation is handled with learning functions. There is no global network effect, firm's products are incompatible and each customer's network effect value is linearly increasing in the number of buyers in his or her neighborhoods. Installed based is not a consideration for customers (adopters), this information is only valuable for firms using it before price-setting. Each period, firms must decide new rates which differentiate between harvest and invest strategies. Firms try to maximize their profit for a multi-period model. There are fourteen initial conditions which shape output values; total period count, switching cost multiplier, network effect multiplier, autarky value, discount value, network structure, customer locations, customer count, firm count, investment rule, learning parameter, unlearning parameter, learning velocity, unlearning velocity. These conditions are input values for a simulation programming structure which can easily change to observe system reaction.

I have observed different market outcomes when incompatible technologies compete against each other and both network effects and switching costs are essential features of the market.In many instances, competition between technologies leads to a persistent monopoly outcome where one technology becomes de facto standard. In other instances, market sharing outcomes emerges. Some instances shows coordination failure cases and some of them have no equilibrium. Ambiguity and incomplete information about market conditions may change firm competition behaviour that result in various and unpredictable market outcome. The simulation has shown that market conditions in a network effect and switching cost which does not only depend on the set of pricing

[6]Heterogeneity is contrasted with the case of a representative agent model in which all agents are assumed to be identical. Economic models often use uniformity (homogeneity). In the real world, every agent has a different behavior pattern and cognitive capabilities. The agents have adaptive expectations rather than rational expectations. Representative agent methods are not used in agent-based models. Agent based computational economics takes account of based upon cognitive, social and individual preferences.

[7]Agents in real world have neither infinite global information nor infinite computational power. Thus agent based modelling assumes local information and bounded rationality.

[8]Agents interact with other agents like neighbors, classmates, etc. Local information is intensive then global information in agent based modelling network structures is often used.

strategies chosen by competing vendors but also strongly depends on the topological structure of the customers' network. This expresses the inappropriateness of installed base models.

2 Model

In this section, I consider an unconventional version of the standard textbook model of switching cost and network effect together. There is a finite set of n consumers indexed by $n = 1, ..., N$, and products which can be supplied by two firms ($k = 2$). These products are ex ante undifferentiated and functionally identical but after the purchase of one of them by a customer, they become differentiated by switching costs and network effect. Agents have to make new decisions to maximise their aggregate utilities or profits each period by considering new situations. In simulation model, maximizing situations depend on some ambiguous circumstances. For example, firms can not be conscious of network structure in which their relevant or potential customer's location are included and the customers cannot easily determine consuming decisions since there is no announcement for future prices. This information deficiency can not always handled with expectation operators so that conventional analytical solutions are going to be failed.

This paper models interaction between agents by means of a graph where each node represents customers. In this manner the customer utility depends partly on the number of his/her neighborhood rather than the total number of customers. In the customer utility function, there is a synchronization part that represents neighborhood's consumption preferences known as local network effect.

Firms focus on their market base and future profits without knowing who their existing and potential customers are within local network structures Customers are greatly influenced by local network effect and switching cost as well as product's inherent qualities. Customers cannot easily change their suppliers by only considering prices [9] and qualities. This situation is known as customer lock-in which makes a customer dependent on a firm for products and services. In oligopoly market structure firms use the customer lock-in factor to negotiate with their customers to get better deals .

The figure 1 shown below, is a basic representation of the model designed. Customers interact with each other and this interaction shapes consumer preferences. Independent of network structure, firms can only use market share information.

Each period firms announce prices which are evaluated by customers to choose the most valuable product or service. Following customers' evaluation process, firms' market shares are determined spontaneously. The period after, firms use this market share information to generate their price strategy. If firms' price level is sufficiently low compared to its rival when switching costs exist the firm can take some percentage of market share (in some situations the firm may dominate market). At the instant time t, Firms' market share definition represented as;

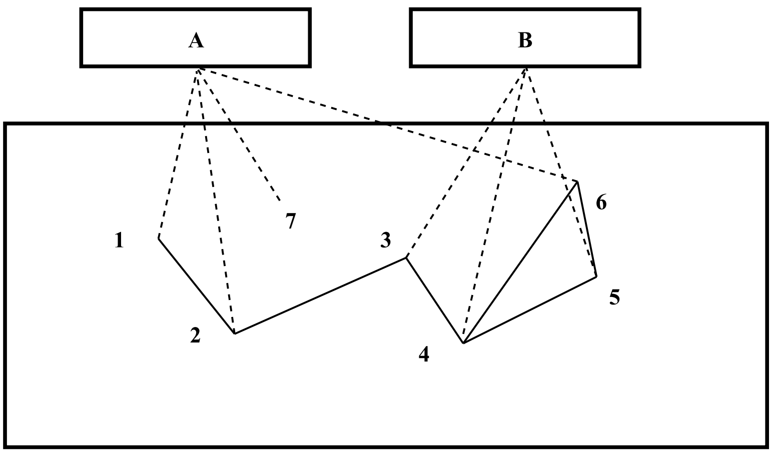

Figure 1: Representation of the model

[9] Firms can announce just one price for all customers.

$$\sigma_i^0 \in [0,1] \qquad i \in (A,B) \qquad k=2 \qquad \sum_{i=1}^{k} \sigma_i^0 = 1 \qquad \sigma_A^0 + \sigma_B^0 = 1.$$

Both location settings and local network structures could make customers be heterogeneous. The location vector holds the distance value from firms which are specified with vectors. Location vectors are generated randomly at the beginning of the simulation and assigned to customers. In real world example, the distance value introduces confidence, loyalty, advertisement effect, firm's prestige, ease and convenience of buying products and services, consuming routines, acquired information and habits by using product or service etc. The model encapsulate these characteristics by using distance value thereby it can get close to real world structure. I use distance value dynamically that is updated every period.

At the instant time t, Customer j has a location definition which is represented as $x_{j,t}$ (Figure 2).

$$j \in \{1,..,m\} \qquad x_{j,t} \subset \Re^k \qquad k=2$$

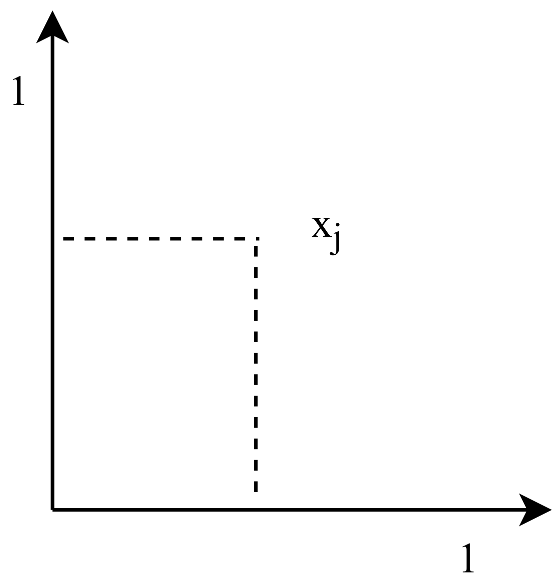

Figure 2: Location of Customer j

Customer's location is one of the initial condition which is set randomly before simulation starts. This variable is updated by product usage at the end of the period so that it can be used with updated value at next period. Customers get extra information about product or service by using it. Product usage is the process in which usage increases loyalty through learning effect in the model. I assume that learning curve is concave and decreasing. Customers get more information about the product during initial periods rather than latter ones. On the contrary, customers lose experience when they do not use firm's product or services and I assume unlearning curve is convex and increasing. The structure described above is realistic for high tech industries. The learning-curve is important in strategic planning since it means that increasing a market share could also bring advantages in competition.

x period counts that customer use relevant products or services over and over (sequentially), LV customer learning velocity, ULV customer unlearning velocity, LP customer learning parameter and ULP represent customer's unlearning parameter. Respectively, customer learning and unlearning functions are shown below[10];

$$\text{Customer Learning Function: } LP(x^{LV} - (x-1)^{LV})$$

[10]These parameters are set for initial conditions like; $LV:1$, $ULP:1$, $LV:0.5$, $ULP:1.1$. This values can be easily changed for different sectors and market conditions.

$$\text{Customer Unlearning Function: } ULP(x^{ULV} - (x-1)^{ULV} - 1)$$

There is a relationship between switching costs and location distance in the model. A total value of switching costs is defined as below;

$$\phi_i(1 - x_{j,t}\vec{e}) \qquad i \in (A, B, ..) \qquad e \in (i, j, ..) \qquad (e \text{ is the axis of a cartesian coordinate system})$$

ϕ_i is a multiplier of location distance that might be assigned different values for different firms [11]. ϕ_i symbolize firm's characteristics. For real world example, ϕ_i might be bureaucratic procedures, infrastructure level, marketing, selling and after selling procedures etc.

Customer's utility function contains local network effect. j customer index, i firm index and t represents current period.

$$\text{Local Network Effect: } \beta_{j,i,(t-1)} \in [0, 1]$$

I denote customer's first degree neighborhood count by $s(x_j)$ and the count of first degree neighborhood that adopt same product or service by $s(x_{j,i,t})$. For my analysis it is convenient to define the ratio of $s(x_j)$ to $s(x_{j,i,t})$ by

$$\beta_{j,i,t} = \frac{s(x_{j,i,t})}{s(x_j)}$$

γ_i is a multiplier of local network effect that might be assigned different values for different firms. γ_i refers to advantage of being in a network such as its popularity, prestige and loyalty etc. These kind of properties can be easily changed for different sectors and market conditions like learning effect or switching cost.

$$\text{Total local network effect: } \gamma\beta_{j,i,(t-1)}$$

All consumers have valuation of the stand-alone value of the products, ϑ. If this variable is sufficiently high, the market is always covered such that all customers are motivated to buy product or service from one of the suppliers because of the positive utility value [12]. Size of the variable ϑ is important for the decision of consumption. If this variable is not sufficiently high, there might be a negative utility value due to high prices and switching cost values resulting in lack of consumption. ϑ is one of the initial conditions in the simulation that different ϑ values create varied aggregate market outputs.

ϑ, switching costs, local network effect and prices are the main parts of the customer utility function. This function can be written as below;

$p_{i,t}$ represents firm i's price at the period t,

Customer j utility function is;

$$U_{j,i} = \begin{cases} \vartheta + \gamma_i\beta_{j,i,(t-1)} - p_{i,t} \qquad if \qquad (i)_t = (i)_{t-1} \qquad i \in \{A, B, ..\} \\ \vartheta + \gamma_i\beta_{j,i,(t-1)} - p_{i,t} - \phi_i(1 - x_{j,t}\vec{e}) \qquad if \qquad (i)_t \neq (i)_{t-1} \qquad i \in \{A, B, ..\} \end{cases}$$

The firm's profit function is;

Firm i's Marginal cost function; c_i and m is the number of customer of relevant firm.

$$\pi_{i,t} = m(p_{i,t} - c_{i,t})$$

I suppose the firm's cost function is the same [13]; which is $c_i = c, \quad i \in \{A, B, ..\}$

I use Farrell and Klemperer's (2007)' s notation that shows firm's i current-period value function (i.e., total discounted future profits), $V_{i,t}$, as the sum of its current profits, $\pi_{i,t}$, and its discounted next-period value function $\delta(V_{i,(t+1)}(\sigma_{i,t}))$, in which σ is the discount factor and the next-period value function, $\delta(V_{i,(t+1)}(.)$, is a function of the size of its current-period customer base, $(\sigma_{i,t})$. In general, the firm's future profits depend on its customers' types and their full histories, expectations, how market share is distributed among competing

[11] I assigned $\phi_i = \phi$ in the model.

[12] One of the assumption is customers can consume at most one product.

[13] A constant value is used as the value of c_i to calculate utilities as cardinal numbers

firms, how many consumers in the market make no purchase, etc. but in relevant literature market share is very often used.

$$V_{i,t} = \pi_{i,t} + \delta(V_{i,(t+1)}(\sigma_{i,t}))$$

As Equations illustrate, the firm must balance the incentive to charge high prices ("harvest strategy) to get greater current profits against the incentive for low prices ("invest strategy") that get higher market share and hence increase future profits.

The firm's (i) first-order condition for the optimal choice of a period-t price is

$$\frac{\partial V_{i,t}}{\partial p_{i,t}} = \frac{\partial \pi_{i,t}}{\partial p_{i,t}} + \delta \frac{\partial V_{i,(t+1)}}{\partial \sigma_{i,t}} \frac{\partial \sigma_{i,t}}{\partial p_{i,t}} = 0$$

The main results are;

$$\frac{\partial \pi_{i,t}}{\partial p_{i,t}} > 0$$

$$\frac{\partial \sigma_{i,t}}{\partial p_{i,t}} < 0$$

Firms have two main strategy, one of them is paying more attention to get current period profits (harvesting) and other is taking care of high market share (investing) in order to increase future profits. The tradeoff between harvesting and investing depends on interest rates, the state of the business cycle, expectations about customer profiles, rival strategy behaviours, market network structure, regulation rules, exchange-rates, market shares, firm profiles and other macroeconomic aggregates (such as GDP or total employment) etc. Some of these dependent cases like rival strategy behaviours, market network structure can be handled by this model. Harvesting and Investing strategies (H,I) does not indicate single point like p_i on price interval $(0 < p_i < \infty)$. In the model these strategies specify an interval like $(\underline{p_i} < p_i < \overline{p_i})$ The intervals for investing and harvesting strategy are represented below;

Investing strategy (I); $[c - (\phi + \gamma), c + \gamma]$

Harvesting strategy (H); $[c, c + \phi + \gamma]$

These two strategy's price sets have common members. The intersection of Investing strategy and Harvesting strategy, is $[c, c + \gamma]$. This unusual situation arise from heterogeneous agents which have different level of switching cost and network effect. These two phenomenon affect customer's utility function in different directions. Looking from the viewpoint of simulation perspective, these interval is useless for price selection by firms, hence model needs one more extra variable to use point prediction. Firms should determine a point that refers value of price in relevant interval which makes the best profit. Variable α determines point gaps in interval which are tested by simulation program.

For strategy I; If $\alpha = 0.5$ $(\alpha \in [0, 1])$;

Firm i 's price; $p_{i,t} = c - (\phi + \gamma) + \frac{\phi + 2\gamma}{2}$

If a realistic analysis is tried to model Bertrand duopoly competition in markets with network effects and consumer switching costs, a question arises as to the firm's future price preferences and profits. Since the answer can not be given by analytical approach, I have developed a simulation generated by agent based computational techniques, and analyzed the results for different initial conditions and different price alternatives. Literature mostly argue that switching cost and network effect increase price levels. I have tried to develop a more flexible model to check if this statement is valid or not. Similar to flexible assumptions, model's fiction and the sequence of the process are very important too. There are three stages in each period;

1. Prices are simultaneously set by firms and announced to every agent in the market. Firm's main inputs when deciding price level are market shares and past profit levels. $(t \neq 0)$

2. Customers evaluate these prices and use as an input for their utility function. After that customers choose most valuable product or services.

3. After consumption, firms calculate their current market share and total profits together with discount value. These two inputs are used to decide price level for the next period (item 1)

One of the objective of the model is to analyze potential coordination failures. For this purpose, After all price announcements, model save equilibrium, nonequilibrium and multi equilibrium situations. Potential price set (potential announcements) for each firm determined by α. For example if $\alpha = 0.05$ then there are $1/\alpha + 1 = 21$ potential announcements for each firm and simulation program analyzes every price announcements in the market. That shows which pair of price or prices are the best choice. Additionally mixed strategy equilibrium is calculated for the price preferences. Systems output values are listed below;

Outputs

1. Maximum Value
2. Minimum Value
3. Average Value (Mix Strategy Equilibrium)
4. Positive Value Percentage
5. Maximum Total Value (Strategy couple)
6. Minimum Total Value (Strategy couple)
7. Nash Equilibrium Values

This result is just get for one element of initial conditions subset. System's initial conditions are presented below;

Initial Conditions

1. Network Structure
2. Customer Locations
3. Customer Count
4. Firm Count
5. Investment Rule
6. Total Period
7. Gamma γ
8. Phi ϕ
9. Discount Value δ
10. Default Utility Value (autarky value) ϑ
11. Learning Parameter
12. Learning Velocity
13. Unlearning Parameter
14. Unlearning Velocity

3 Simulation

The simulation model [14] described in the preceding section is designed as a multi-step architecture. In the first step of the process, a module named as "Orchestration", fetches all combinations of initial conditions stored in a relational database. There are two more input variables besides initial conditions. One of them is called "alpha" [15] defined according to how many equal parts the strategy interval is divided. Other variable is called "thread" and defined according to how many simulation processes is executed concurrently [16] . Text below, shows a pseudocode of the orchestration module.

```
    Declare a String variable called "alpha" and initialize it;
Declare a String variable called "thread" and initialize it;
Get all conditions from database which is declared before;
For each condition,
     Call "Alpha Organizer Module" with alpha, thread and current condition
variables;
     Call "Reporting Module";
```

Orchestration Module is executed for each element in the cartesian product of the sets of initial conditions. There are two main parts in this code block. In the first part, a module named as "Alpha Organizer", is called. This module controls "Core Simulation Module" which is executed $\alpha^{firmcount}$ times. Text below, shows a pseudocode of the Alpha Organizer Module

```
    For each alpha for firm 1,
     For each alpha for firm 2,
         For each alpha for firm 3,
             Call "Core Simulation Module" with alpha, thread and current
condition;
```

For example, lets analyse potential price selections for firm count, k = 3 and alpha, $\alpha = 20$. Each firm has to choose its price level in a set of prices which has 21 distinct values. Startpoint of definition interval for invest and harvest strategies represent as $\bar{x}$ and interval lenght represents as y. The firms prices are;

$$p_a = \bar{x} + \frac{y}{\alpha}\alpha_a,\ p_b = \bar{x} + \frac{y}{\alpha}\alpha_b \text{ and } p_c = \bar{x} + \frac{y}{\alpha}\alpha_c$$

$\alpha_a, \alpha_b, \alpha_c \in [0, \alpha]$, One of the instance of this formulation is;

$$p_a = \bar{x} + y0.05,\ p_b = \bar{x} + y0.8 \text{ and } p_c = \bar{x} + y0.35$$

Total count of price combinations for these three firms is $(alpha + 1)^k = 21^3 = 9261$ and Core Simulation Module is called 9261 times for each combination of the initial conditions.

Core Simulation Module checks all of potential strategy combinations for the entire period so that all potential output values can be observed. These output values are written to a specific file parsed by Reporting Module later. Text below, shows a pseudocode of the Core Simulation Module.

[14] All parts/modules of the application are written in java programming languages (https://www.java.com)

[15] The interval of the investment strategy is $[c-(\phi+\gamma), c+\gamma]$ and the interval of the harvest strategy is $[c, c+\phi+\gamma]$. A firm which choses one of these strategies, should specify a price level to announce and that represents a point in relevant strategy interval. The model force agents (firms) to elect a price level which is a starting point of one of the fragment that defined by alpha value. This finite selection model is requisite for simulation. If the fragment count raises (higher alpha value), the model's expressing capability increases but the cost of model increase in the same time. There is a trade off between the cost and the expressing capability.

[16] The cost expressed above can be diminished by multithreading in computer architecture. This is a one of the optimization property for programming languages.

```
    Initialize all conditions;
Calculate Strategy Matrix;
For each strategy couple (firm1 & firm2),
    For each period,
        Announce Prices.
            Calculate utilities and profits for all object.
                Calculate firm's customer base.
Create "report file".
```

When core simulation module starts, firms define their strategy set which has 2^{τ} elements (two strategies; harvest - invest and τ represents period count). As an illustration, a character sequence like "0101101011" represents strategies for 10 periods. The index of character in the representation, defines strategic decision in the period which has same index. "0" refers to strategy of investing and "1" refers to strategy of harvesting in relevant period. Pay-off matrix (Strategy Matrix) is created with $(2^n)^k$ cells. Firms announce their prices for each period, then customers prefer one firm considering switching costs, network effects and prices. These preferences affect firm's profits and customer bases which are an input for next period profits.

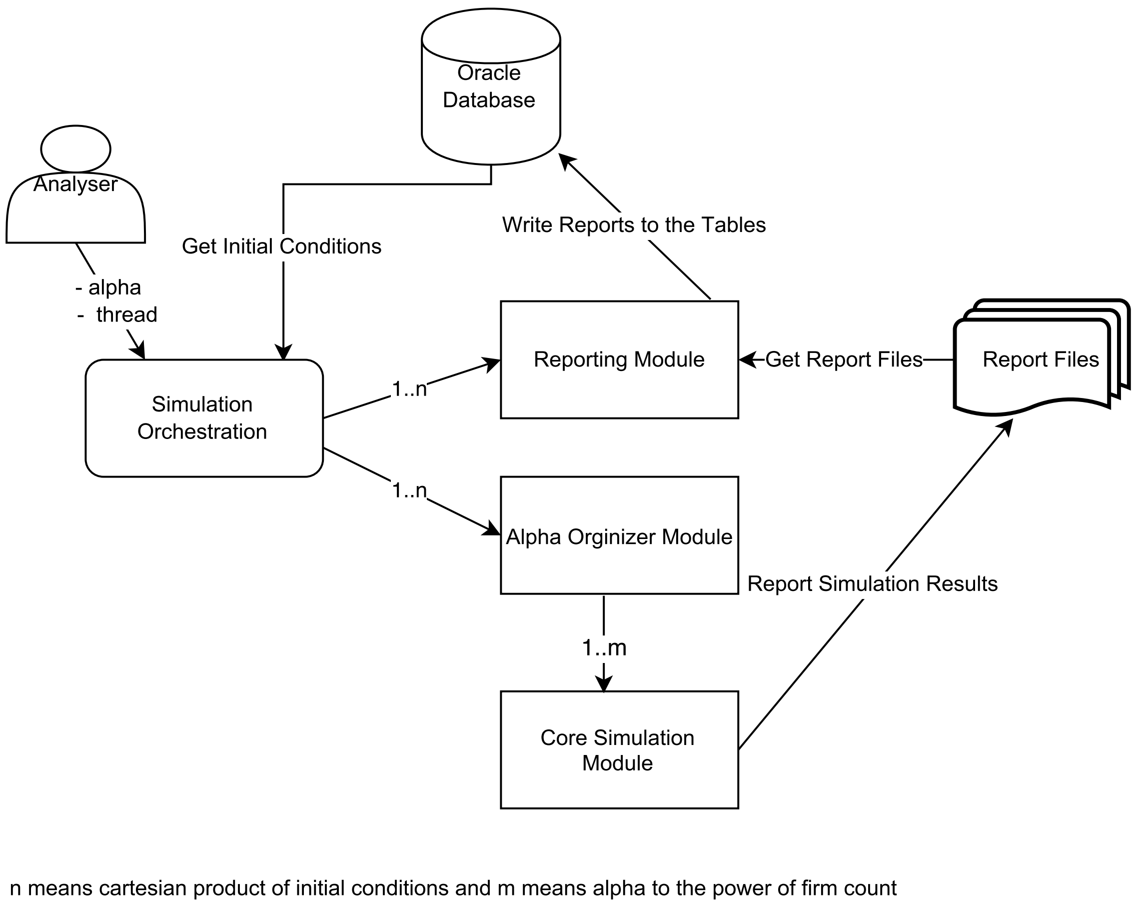

Figure 3: General Architecture

In the second part of the pseudo code (orchestration module), a module named as "Reporting", is called. Output files which are created by core simulation module, are parsed by Reporting Module. Then reporting module inserts them to database tables. These entries are used for reporting and monitoring system outputs.

Core Simulation Module

Core simulation Module is the most important part of the simulation ecosystem. All agents interact with each other, and their utility functions include this interactions both directly and indirectly. Network effect and location values exist directly in customer's utility functions. Customer's choices define firm's customer base that affect the price decisions at the next period. Agents have lots of ambiguities (incomplete information) which may lead to inconsistent decisions. The model shows potential decisions in the system that customers and firms are affected by switching costs and network effect.

First of all, I try to set out heterogeneous agents types (customers and firms). Then general process will be illustrated.

Customers
Below is the customer properties with descriptions;
Customer Variables;

1. *firmDistance:* This variable holds distances between customers and firms.
2. *firmMap:* This Map holds firm specific values that customers know as public information.
3. *firmBasedUtulityValuesMap:* Customers evaluate switching costs, network effect, location, learning effect and prices etc. to form utility value for each firm. These utility values are held to compare which utility value is greater than others.
4. *friends:* In this simulation model rather than global network effect, local network effect is prefered by this way customer's neighbor's consumptions are important. This variable holds neighbors of current customer.
5. *friendDistribution:* This variable holds usage percentage about each product among customer's friends.
6. *totolFriendCount:* Customer's total friend count
7. *location:* This variable defines customer location.
8. *stepCount:* it holds current period index.
9. *currentFirm:* This variable holds current consumption preference.
10. *firmPeriodCount:* It holds how many period customer consume same product over an over (measure of loyalty) and how many period he/she doesn't consume it. This variable is used in learning function.
11. *id:* Customer Identity

When simulation starts, first, *firmMap* and *id* variables are initialized for each customer. *Location* variable is initialized by initial condition then *firmDistance* is filled by using location value. Each period *step()* function is called. This function manages all customer operations such as calculating utility function. Pseudocode is presented below;

```
    step{
If stepCount = 1 then initialize friends and totolFriendCount by using
initial conditions.
initialize friendDistribution variable.
For each firm;
    calculate switching costs.
    calculate network effect.
    calculate utility value by using switching costs, network effect,
announced price, consume value.
Chose best utility value.
Consume product and fill currentFirm variable.
Update location information by using learning effect.
}
```

Firms
Below is the firm properties with descriptions;
Firm Variables;

1. *id:* Firm Identity.
2. *location:* "Location" variable use for firm location. This location information is constant during simulation.
3. *marjinalCost:* Marginal cost of firm.
4. *currentPrice:* Announced price at current period.
5. *currentStrategySet:* Assigned strategy set at current period.
6. *stepCount:* it holds current period index.
7. *base:* Firm's current period market share.

Firms have two main functions, named as *announcePrice* and *setFitnessValueAfterStep* respectively. *announcePrice* method is called at the beginning of the period and *setFitnessValueAfterStep* method is called at the end of the period. *announcePrice* method determines period prices by using *currentStrategySet* and *alpha* values. After customers make their decisions by these announced prices, *setFitnessValueAfterStep* method is called. This method is used to calculate total output values for current strategy set.

Mechanism of Core Simulation Module

An instance of the combination of the initial conditions and alpha value are the input values of the core simulation module. This module executes *optimizeAlpha* method to calculate output values and report them for all instances of strategy set.

```
   optimizeAlpha{
Create Strategy Matrix;
For each cell in Strategy Matrix;
    Create Firms and keep account of them to firmMap;
    Create Customers with location, network structure and utility function.
For each Period;
        Firms Announce Prices (announcePrice function in firm class)
        Customers calculate utility value by using switching costs, network
effect, announced price, consume value and chose best utility value (step
function in customer class)
        Firms calculate their profits and market shares and write it to
relevant cell on strategy matrix (setFitnessValueAfterStep function in firm
class)
Call Report Function
}
```

4 Results

The model described above, agents make choices at the same period in mutual awareness of each other. They are characterized by different information sets and they have no meaningful pattern for choosing absolute strategy in non-repeating game such that expectation operator becomes useless. Discriminating between complete information case and incomplete information case is representing differences between formal analyses and complex analyses in the simulation model. Output levels could show differences between information degrees.

In the complete information perspective, I suppose that each agent knows other agents' all information and properties and perfect cognitive abilities so that they can easily maximize their utulities and profits. This assumption is consistent with formal literature. In addition I relax lots of assumptions like interaction with agents. As for the incomplete information perspective, cognitive abilities come into prominence that makes model more difficult to solve.

Simulation Results in Complete Information

All the subgame equilibrium values' sign [17] is calculated positive ($\phi >= 0$) at the end of the simulation process ,which is similar with literature. The percentage of 0 ($\phi = 0$) values from all equilibrium values is 29% . 0 profit level is not significant for firms so they do not enter into the market under this condition. If there is no subgame equilibrium for relevant alpha couple, mixed strategy equilibrium is calculated. All of these equilibrium values (both nash and mixed strategy equilibriums) which are calculated from subgames, generate main payoff matrix that gives the possible outcome of a twofirm. The row of the strategy (possible alpha values for firm 1) is chosen by firm 1 and the column of the strategy (possible alpha values for firm 2) is chosen by firm 2. Strategic dominance occurs when one alpha strategy is better than another alpha strategy for one firm, no matter how that player's opponents may play. The table above (Table 1) symbolize this situation.

Table 1: Matrix presentation of alpha values

Strategies	Firm 2 Alpha 1	Firm 2 Alpha 2
Firm 1 Alpha 1	$\underline{5}$,5	$\underline{2}$,$\underline{8}$
Firm 2 Alpha 2	3,$\underline{3}$	1,2

For example, the simulation results for report $id = 3158$ [18] , has 121 ($11 * 11$) different alpha couple alternative which are placed to payoff matrix. Each cell represents subgame which is calculated by core simulation module. The cell values are sub game nash equilibrium(s) or mixed strategy equilibrium. Unique subgame nash equilibrium count is 97 (Table 2).

Table 2: Subgames which have unique equilibriums for alpha couples (Report $id = 3158$), form a part of main payoff matrix.

ReportAlpha	Reportid	Strategy1	Strategy2	Value1	Value2
1;1;	3158	00100	00100	301.5	238.5
1;0;	3158	00100	11111	0	0
1;0.9;	3158	01111	00101	685.89	18.9
1;0.8;	3158	00110	01111	1.8	610.2
1;0.7;	3158	00100	01111	0	513
1;0.6;	3158	00100	01111	0	414
1;0.5;	3158	11111	00101	350.1	101.7
1;0.4;	3158	11111	00111	344.88	18.9

[17] These are sub game equilibrium values which are results for each alpha couple.

[18] Initial conditions for report $id = 3158$ represented below;

"REPORTID" "KEY" "VALUE"
3158 "CONNECTION_PROBABILTY" "0.3"
3158 "DELTA" "1"
3158 "GAMMA" "0.9"
3158 "INVEST_RULE_COUNT" "3"
3158 "LEARNING_PARAMETER" "1.1"
3158 "LEARNING_VELOCITY" "0.5"
3158 "LOCATIONS" "0.7767832587644451;0.27590643373027923;....
3158 "NETWORK" "1-3;1-6;1-7;1-8;1-9;1-10;....
3158 "NUM_CUST" "100"
3158 "NUM_FIRMS" "2"
3158 "PHI" "0.9"
3158 "PRODUCTVALUE" "100"
3158 "TOTAL_PERIOD" "5"
3158 "UNLEARNING_PARAMETER" "0.9"
3158 "UNLEARNING_VELOCITY" "1.2"

1;0.3;	3158	01111	11111	6.3	266.22
1;0.2;	3158	11111	01110	179.28	1.8
1;0.1;	3158	00100	11111	0	90
0;1;	3158	11111	00100	0	0
0;0;	3158	11111	11111	0	0
0;0.9;	3158	11111	01100	0	1.26
0;0.8;	3158	11111	00100	0	6.12
0;0.7;	3158	11111	00110	0	6.84
0;0.6;	3158	11111	01111	0	19.08
0;0.5;	3158	11111	00111	0	127.35
0;0.4;	3158	11111	01111	0	216
0;0.3;	3158	01111	11111	117	0
0;0.2;	3158	11111	01111	0	18
0;0.1;	3158	11111	11111	0	3.42
0.9;1;	3158	00100	01111	31.5	669.06
0.9;0;	3158	01001	11111	0.63	0
0.9;0.9;	3158	01001	01001	251.37	261.63
0.9;0.8;	3158	00110	01111	30.15	564.12
0.9;0.7;	3158	00100	01111	1.26	511.65
0.9;0.6;	3158	01111	00100	414	0
0.9;0.5;	3158	00100	01111	0	315
0.9;0.4;	3158	00100	11111	99.81	252.72
0.9;0.3;	3158	00100	11111	30.87	243.54
0.9;0.2;	3158	01001	11111	6.93	176.04
0.9;0.1;	3158	11111	01010	89.1	3.15
0.8;1;	3158	01111	00100	608.4	3.6
0.8;0;	3158	00100	11111	2.88	0
0.8;0.9;	3158	01111	00101	501.12	65.07
0.8;0.6;	3158	00111	01111	2.16	411.3
0.8;0.5;	3158	00100	01111	0	315
0.8;0.4;	3158	01111	00100	216	0
0.8;0.3;	3158	00100	01111	0	117
0.8;0.2;	3158	00111	11111	38.88	141.12
0.8;0.1;	3158	11111	00111	85.86	8.28
0.7;1;	3158	01111	00100	513	0
0.7;0;	3158	00110	11111	7.38	0
0.7;0.9;	3158	01111	00111	507.6	5.04
0.7;0.5;	3158	01111	01001	315.9	0.18
0.7;0.4;	3158	01111	00100	216	0
0.7;0.3;	3158	00100	01111	0	117
0.7;0.2;	3158	00100	01111	0	18
0.7;0.1;	3158	11111	00100	64.62	13.86
0.6;1;	3158	00100	01111	0	414
0.6;0;	3158	10010	11111	0	0
0.6;0.9;	3158	01111	00100	414	0
0.6;0.8;	3158	00100	01111	1.44	412.2
0.6;0.4;	3158	00101	01111	0	216
0.6;0.3;	3158	01111	00100	117	0
0.6;0.1;	3158	11111	00111	0.72	288.72
0.5;1;	3158	11111	00111	332.1	120.6
0.5;0;	3158	01011	11111	106.2	0
0.5;0.9;	3158	01111	00100	315	0

0.5;0.8;	3158	01111	01110	315.45	0.36
0.5;0.7;	3158	01001	01111	0.27	316.35
0.5;0.2;	3158	01111	00100	18	0
0.5;0.1;	3158	11111	01111	0.9	311.85
0.4;1;	3158	11111	00101	338.4	27
0.4;0;	3158	01111	11111	216	0
0.4;0.9;	3158	01111	00100	216	0
0.4;0.8;	3158	01111	00100	216	0
0.4;0.7;	3158	01111	01100	216.72	0.09
0.4;0.6;	3158	00110	01111	0	216
0.4;0.5;	3158	10101	00101	0	45
0.3;1;	3158	11111	00110	264.6	9
0.3;0;	3158	01111	11111	117	0
0.3;0.9;	3158	11111	00101	228.96	47.88
0.3;0.8;	3158	01111	00100	117	0
0.3;0.7;	3158	01111	00100	117	0
0.3;0.6;	3158	01111	00100	117	0
0.3;0.5;	3158	10101	00101	0	45
0.3;0.4;	3158	11011	01011	0	72
0.3;0.1;	3158	11111	01111	0	117
0.2;1;	3158	11111	00100	178.92	2.7
0.2;0;	3158	01111	11111	18	0
0.2;0.9;	3158	11111	01111	172.44	13.23
0.2;0.8;	3158	11111	00101	138.24	41.76
0.2;0.7;	3158	00100	01111	0	18
0.2;0.5;	3158	01111	00100	18	0
0.2;0.1;	3158	11111	01111	0	18
0.1;1;	3158	11111	00110	89.82	0.9
0.1;0;	3158	11111	11111	1.44	0
0.1;0.9;	3158	11111	01010	88.92	3.78
0.1;0.8;	3158	11111	00100	80.46	19.08
0.1;0.7;	3158	00100	11111	42.48	51.84
0.1;0.6;	3158	11111	00111	0.36	288.36
0.1;0.5;	3158	01111	11111	315	0
0.1;0.3;	3158	01111	11111	117	0
0.1;0.2;	3158	11111	01111	0	18
0.1;0.1;	3158	11111	11111	42.3	47.7

Multi subgame nash equilibrium count is 11 (Table 3).

Table 3: Subgames which have multi equilibriums for alpha couples (Report $id = 3158$), form a part of main payoff matrix.

ReportAlpha	Reportid	Strategy1	Strategy2	Value1	Value2
0.8;0.8;	3158	01010	01010	194.04	201.96
0.8;0.8;	3158	01001	01001	190.08	205.92
0.8;0.8;	3158	00100	00100	144	144
0.7;0.7;	3158	01001	01001	125.55	153.45
0.7;0.7;	3158	00100	00100	72.9	89.1
0.6;0.6;	3158	01001	01001	59.94	102.06
0.6;0.6;	3158	00101	00101	77.76	84.24
0.5;0.5;	3158	10101	10101	91.8	88.2

0.5;0.5;	3158	01101	01101	86.4	93.6
0.5;0.5;	3158	00101	00101	17.55	27.45
0.4;0.4;	3158	11011	11011	95.04	120.96
0.4;0.4;	3158	01101	01101	33.84	38.16
0.4;0.4;	3158	01011	01011	33.12	38.88
0.4;0.2;	3158	11011	01011	0	72
0.4;0.2;	3158	00111	10111	72	0
0.4;0.1;	3158	11111	01111	0	216
0.4;0.1;	3158	11101	01101	0	72
0.4;0.1;	3158	01011	11011	72	0
0.4;0.1;	3158	10111	00111	0	72
0.3;0.3;	3158	11111	11111	140.4	129.6
0.3;0.3;	3158	10111	10111	49.14	67.86
0.2;0.4;	3158	01101	11101	72	0
0.2;0.4;	3158	11011	01011	0	72
0.2;0.2;	3158	11111	11111	97.2	82.8
0.2;0.2;	3158	01111	01111	9.9	8.1
0.1;0.4;	3158	11111	01111	0	216
0.1;0.4;	3158	11101	01101	0	72
0.1;0.4;	3158	11011	01011	0	72
0.1;0.4;	3158	10111	00111	0	72

Mixed subgame equilibrium count is 13 (Table 4) .

Table 4: Subgames which have just mixed equilibriums for alpha couples (Report $id = 3158$), form a part of main payoff matrix.

ReportAlpha	Reportid	AvgValue1	AvgValue2
0.8;0.7;	3158	76.9	261.94
0.7;0.8;	3158	248.33	88.37
0.7;0.6;	3158	55.22	149.18
0.6;0.7;	3158	136.93	64.21
0.6;0.5;	3158	25.07	48.45
0.6;0.2;	3158	11.73	-171.94
0.5;0.6;	3158	43.97	28.67
0.5;0.4;	3158	-12.79	-37.17
0.5;0.3;	3158	-15.27	-95.49
0.4;0.3;	3158	-58.35	-114.25
0.3;0.2;	3158	-108.62	-187.3
0.2;0.6;	3158	-172.06	11.91
0.2;0.3;	3158	-187.63	-109.73

All of these alpha couples form a part of main payoff matrix then nash equilibrium(s) are solved. For Report $id = 3158$, I have found unique nash equilibrium which is (Alpha=0.3, Alpha=0.3). The output values for this alpha couple is respectively 140.4 and 129.6

Another analysis can be done via total system output at the end of the simulation. A potential total output interval is represented below for nash equilibrium point (Table 5). The market's output level is between -495 and 270 .

Total count of the combinations of the initial conditions is 1296 and 152 of them has one or more nash equilibrium. One of them whose id is 3133, has multi equilibrium (Table 6)[19] so that there is a coordination

[19] Initial conditions for report $id = 3133$ represented below;
REPORTID KEY VALUE

Table 5: Maximum and minimum values for report $id = 3158$

ReportAlpha	Reportid	MaxSum	MinSum
0.3;0.3;	3158	270	-495

failure potential (The output level of the alpha values which is $0.3; 0.3;$ is higher than $0.5; 0.5;$ level).

Table 6: Nash Equilibriums for report $id = 3133$

ReportAlpha	Reportid	Value1	Value2
0.3;0.3;	3133	133.95	151.05
0.5;0.5;	3133	91.65	103.35

Total count of the combinations of the initial conditions which have no unique or multi nash equilibrium is 1144 and I have calculated mixed equilibrium that resulted in positive values.

Simulation Results in Incomplete Information

If agents have no sufficient information about their environment, location and other agents' preferences, game come to a conclusion on another output level which may be different from equilibrium point which agents have complete information about all of the system properties (Figure 4,5,6,7,8).

In this case, agents try to comment signals from both their environment and other agents. These signals are evaluated by agents experiments. There is also a feedback mechanism which is forced to evolve agents experiments in each step of the game such that interactions plays a major role. The interpretation activity overcome the deficiencies on information set. In formal techniques, agents have either sufficient information or a capability to express an opinion about information patterns. Besides agents have enough cognitive capabilities to make correct decision in rational choices that maximize his/her utility or profit. In general homogeneity is often used. Few study focus on heterogeneity nevertheless they used this concept just for quantitative measurements like a budget not for cognitive values. In this study, generalization process is tried to be much little than formal methodologies like heterogeneity or interaction cases so that the explanatory power of real world issues become more powerful than contemporary ones. The explanatory power can be increased by engineering processes or model add-ons, however cognitive capabilities and levels is a different case to discuss. To model system with evolutionary viewpoint, cognitive issues in other words artificial intelligence must be analyzed. Learning is the key concept for evolutionary cases. Genetic algorithms and machine learning are most useful toolkits for modelling artificial intelligence. Complexity economics is the main discipline that internalize artificial intelligence. This study's basic purpose is to model switching cost and network effect by using these toolkits.

3133 "CONNECTION_PROBABILTY" "0.3"
3133 "DELTA" "1"
3133 "GAMMA" "1.0"
3133 "INVEST_RULE_COUNT" "3"
3133 "LEARNING_PARAMETER" "1.1"
3133 "LEARNING_VELOCITY" "0.5"
3133 "LOCATIONS" "0.7767832587644451;0.27590643373027923;....
3133 "NETWORK" "1-3;1-6;1-7;1-8;1-9;1-10;1-17;1-18;....
3133 "NUM_CUST" "100"
3133 "NUM_FIRMS" "2"
3133 "PHI" "0.9"
3133 "PRODUCTVALUE" "100"
3133 "TOTAL_PERIOD" "5"
3133 "UNLEARNING_PARAMETER" "1.0"
3133 "UNLEARNING_VELOCITY" "1.2"

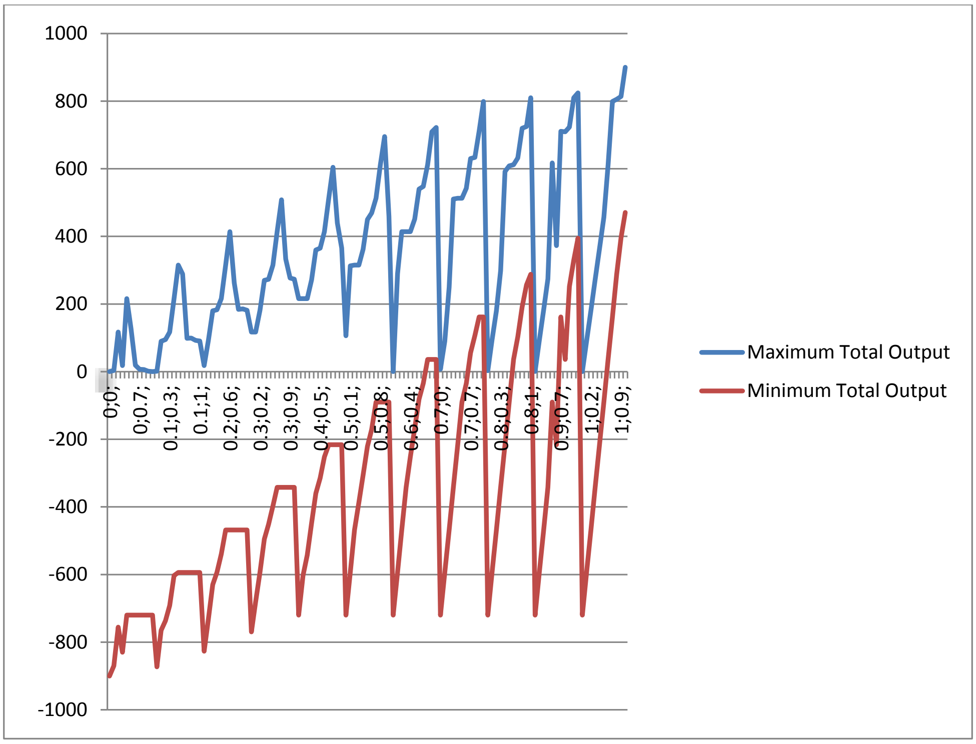

Figure 4: Total profit levels

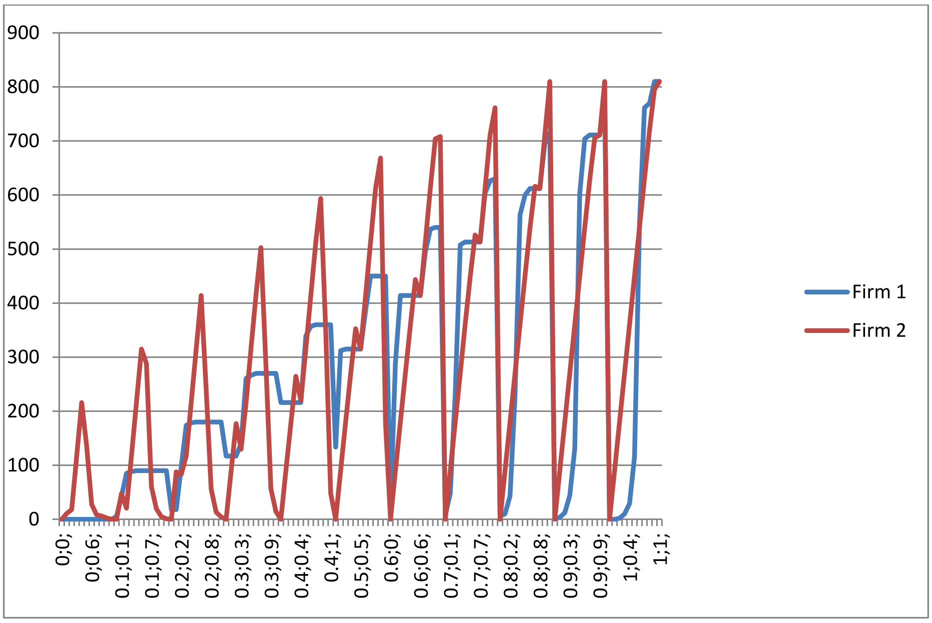

Figure 5: Firm's maximum profits

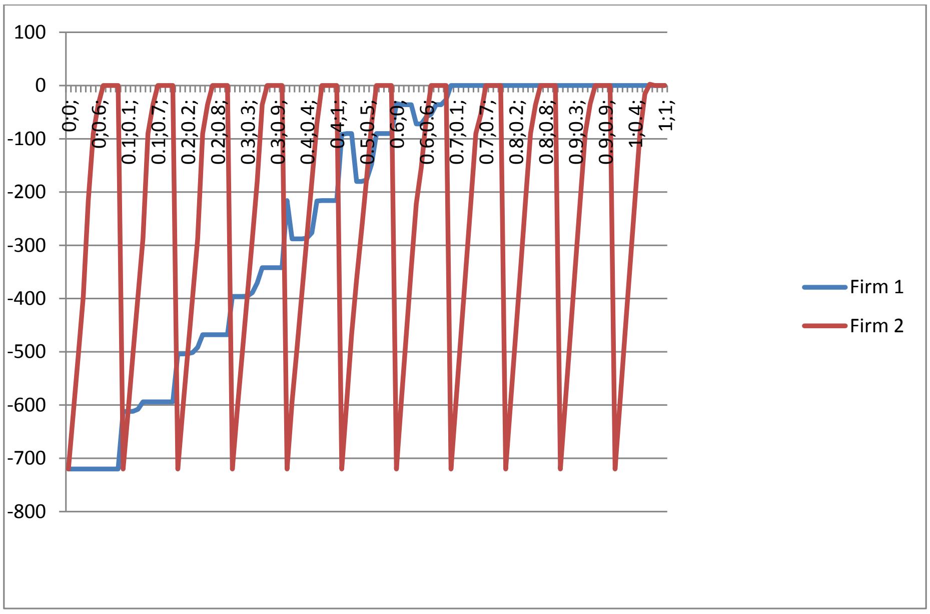

Figure 6: Firm's minimum profits

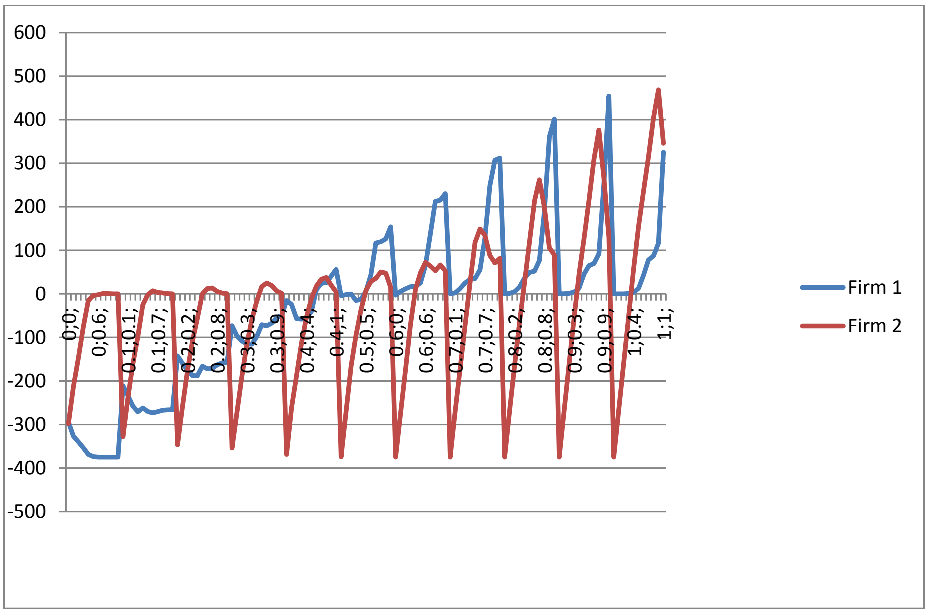

Figure 7: Firm's average profits

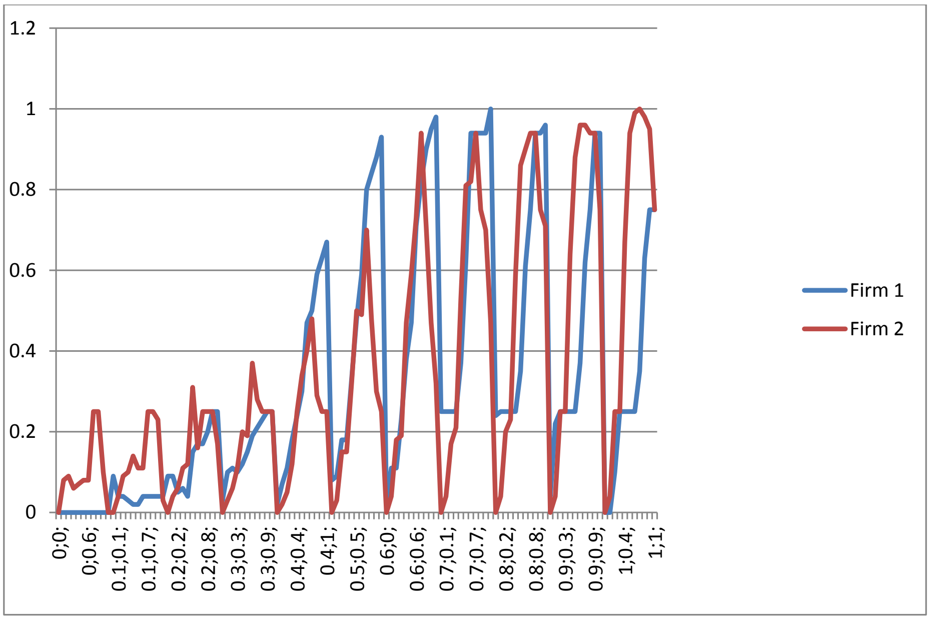

Figure 8: Firm's positive profit value percentages

5 Conclusion

A model of complex systems should contain multi agent structure in which agents interact with each other. Interactions play main role which determine global dynamics and behaviours of systems. Agent based computational methodology presents computational frameworks that permit the study of complex system behaviors. it is very difficult to formally analyze complex systems. The agent based computational model has more explanatory power than conventional analytic solutions because agent based approach makes assumptions less rigid and more realistic. Launbenbacher (2009) pointed out that results obtained through simulations do not formally validate the observed behavior. Thus, there is a need for a mathematical framework which one can use to represent multi agent systems and formally establish their properties and interactions' structure.

This paper presented an agent-based simulation of a complex system by both mathematical framework and computational simulation structure that consists of a collection of agents (firms and customers). This complex system represents market situations in the presence of both switching costs and network effect. The state of an agent at a given point in time is determined through a collection of rules that describe the agent's interaction with other agents. The collection of rules have been presented with formal equations in the model section and the simulation framework have been explained with pseudo codes in the simulations section.

This study aimed to test, critique and comment mainstream theories of the switching costs and network effects, and empirical understanding for how particular observed regularities and irregularities have evolved by using of the methodology of agent-based computational economics (ACE). When the full implications of bounded rationality and complete information assumptions are accepted, a process-based approach is preferred rather than adaptation and evaluation approaches. The process-based approach is sufficiently enough for testing experimentation of mainstream aspect. If unpredictable situations are considered like uncertainties (incomplete information), path dependencies and relationship characteristics between firms and customers, optimal outcomes are not guaranteed. In this case, agent's cognitive capabilities are the main focus point for adapting different situations by agents. This adaptation process measures agent's profit or utility level so that learning algorithms, genetic algorithms and artificial intelligence plays critical role. Even though the integration of both the collective (network effect) and individual (switching cost) dimension in the same framework is a real challenge for cognitive perspective, this study will evolve to this direction to analyze dynamic features of markets viewed as cognitive and complex social interactive systems.

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